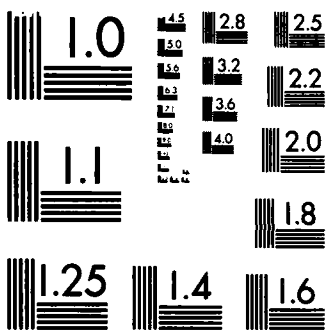
AD-A161 082 DETERMINING OPTIMAL RESOURCEAALLOCATIONS USING 1/2
PRODUCTION MODELS DERIVED F..(U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH SCHOOL OF SYST
UNCLASSIFIED J O MONTEMAYOR SEP 85 AFIT/GLM/LSM-85S-53 F/G 15/5 NL


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6

2

DETERMINING OPTIMAL RESOURCE ALLOCATIONS USING PRODUCTION MODELS DERIVED FROM EFFICIENCY ANALYSIS OF EMPIRICAL DATA

THESIS

Jose O. Montemayor
Captain, USAF

AFIT/GLM/LSM/85S-53

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GLM/LSM/85S-53

DETERMINING OPTIMAL RESOURCE ALLOCATIONS USING PRODUCTION MODELS DERIVED FROM EFFICIENCY ANALYSIS OF EMPIRICAL DATA

THESIS

Jose O. Montemayor
Captain, USAF

AFIT/GLM/LSM/85S-53

The contents of the document are technically accurate, and no sensitive items, detrimental ideas, or deleterious information are contained therein. Furthermore, the views expressed in the document are those of the author(s) and do not necessarily reflect the views of the School of Systems and Logistics, the Air University, the United States Air Force, or the Department of Defense.

AFIT/GLM/LSM/85S-53

DETERMINING OPTIMAL RESOURCE ALLOCATIONS USING PRODUCTION MODELS DERIVED FROM EFFICIENCY ANALYSIS OF EMPIRICAL DATA

THESIS

Presented to the Faculty of the School of Systems and

Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Logistics Management

Jose O. Montemayor, B.S., M.A.

Captain, USAF

September 1985

Preface

This research was undertaken to determine the feasibility of using the information provided by a recently developed methodology for efficiency measurement called Constrained Facet Analysis in specifying the parameters of a resource allocation model. The allocation model sought was one capable of allocating or reallocating resources among a group of related organizations given the following three constraints: (1) a budget, (2) apparent rates of substitution and productivity for each of the organizations, and (3) the goals of production specified by management.

A resource allocation model was developed using concepts from network theory. The model was tried using a contrived data set for a group of 12 tactical fighter wings each of which consumes two inputs, manpower and materiel, in producing two outputs, sorties and mission capable aircraft. The two-input, two-output case was chosen with the hope that the results obtained could be generalized to all other multiple-input, multiple-output situations.

In performing the experiments and writing this thesis, I have had a great deal of help from others. I am deeply indebted to my thesis advisor, Lt Col Charles T. Clark, for his endless patience and technical assistance throughout this effort. I also wish to thank Lt Col Richard L. Clarke and Maj John A. Stibravy who provided invaluable assistance

as readers for this research project. Finally, I wish to thank my family, Dolores, Oscar, Kelly, and Wanda, for their understanding and concern on those many days and nights when I was too busy to show them the affection they so richly deserve.

Jose O. Montemayor

Table of Contents

List of Figures

List of Tables

Page

AFIT/GLM/LSM/85S-53

Abstract

Evaluating the performance of nonprofit organizations and formulating resource allocation policy has long been recognized as a difficult problem for management to solve. This research defines the relationships between the terms efficiency, effectiveness, productivity, resource allocation, and capability as it pertains to military organizations.

This research studied possible ways in which the recently developed efficiency measurement methodology, Constrained Facet Analysis, might be used in solving the resource allocation problem.

The approach taken was that of experimentation with a resource allocation model using a data set that simulated a group of 12 tactical fighter wings each using 2 types of resources, manpower and materiel, and producing 2 types of outputs, sorties and mission capable aircraft days.

The resource allocation model consisted of a generalized network model. Networks have graphic properties which make possible the presentation of the resource allocation problem in nonmathematical terms. Furthermore, the translation of the graphic network model into a mathematical program for computer solution is relatively easy.

The methodology pursued by this research consisted of experimentation with the two-input, two-output case; i.e., given that relative efficiencies and apparent rates of productivity can now be measured among a group of related organizations, should available resources be allocated to increase production to some set level? Or, what is the maximum level of production that can be expected?

The research concludes with recommendations for field testing the resource allocation model using actual data and the help of knowledgeable managers.

DETERMINING OPTIMAL RESOURCE ALLOCATIONS USING PRODUCTION MODELS DERIVED FROM EFFICIENCY ANALYSIS OF EMPIRICAL DATA

I. Introduction

General Issue

According to the honorable Lawrence J. Korb, Assistant Secretary of Defense for Manpower, Installations, and Logistics, "We don't have a good quantitative resource allocation tool that tells us where we should spend our next dollar [to gain further combat readiness]. Currently we use a combination of measurements: war games simulation, senior commanders' qualitative assessments, and static measures of force structure." He concluded that ". . .it is currently not possible to quantify the so-called capability curve [18]."

Ostensibly, methods that help commanders and managers evaluate military capability and efficiency, and formulate resource allocation policy are vitally important to the Department of Defense and the military services. In fact, one of the Air Force manager's primary responsibilities is to use resources as effectively and efficiently as possible in the accomplishment of his or her mission [8]. The public expects the Department of Defense to accomplish national

defense objectives with the lowest possible level of expenditures. In order to develop ways to operate more efficiently, and thereby increase overall capability with the resources that are available, Air Force managers need accurate efficiency performance feedback. Until recently, management's techniques for assessing efficiency have been successful in evaluating only those objectives that they considered most important; i.e., there were no measures of efficiency applicable to the multiple resource, multiple objective situations that are typically found in most Air Force organizations.

Data Envelopment Analysis (DEA) recently developed by Charnes, Cooper, and Rhodes [5], and an extension of DEA called Constrained Facet Analysis (CFA) developed by Clark [6], have made it possible to obtain measures of relative efficiency of organizations. These efficiency measurement techniques also provide information on the possible sources of inefficiency. Both models can simultaneously consider a variety of resources (inputs) used and all of the outputs produced, and can yield a relative measure of efficiency for each of the units evaluated relative to all other organizations in the evaluation group. Additionally, the models yield marginal rates of substitution and productivity from the input and output observations provided. The marginal rates provide information concerning apparent trade-offs among resources consumed and outputs produced in

the most efficient organizations, and might prove to be useful in evaluating alternative resource allocations for the entire group. The concept of input substitution (trade-offs) as defined by economic theory has to do with maintaining some constant level of output through different input combinations. The management task then becomes "to select the particular input combination that minimizes the cost of producing any given level of output [15:145]." Conversely, to maximize the level of output given some set level of resources, management would allocate resources giving priority to those organizations with the highest rates of productivity.

The applicability of Data Envelopment Analysis to measuring the efficiency of nonprofit organizations has been demonstrated in studies of hospitals [20], fire stations [3], schools [4], courts [19], and tactical fighter wings [1]. Constrained Facet Analysis has been applied by the Educational Productivity Council of the University of Texas to evaluate the relative performance of schools [4]. The author proposes to extend the theory and applicability of CFA by developing a structured approach to the resource allocation problem and by exploiting the management information derived from the application of the CFA model.

Statement of the Problem

Evaluating the performance of nonprofit organizations and formulating resource allocation policy has long been recognized as a difficult problem for management to solve [6]. Therefore, this research focused on exploiting the information that results from the application of Constrained Facet Analysis (CFA), and presented an approach to resource allocation using concepts from network flow theory. Specifically, a hypothetical group of organizations (simulating Air Force aircraft wings) was evaluated using CFA. Then a network model was developed for allocating or reallocating resources among the hypothetical group of Air Force organizations given budgetary constraints and organizational goals.

Objectives

1. The primary objective of this research was to develop specific management techniques for dealing with the resource allocation problem through the use of Constrained Facet Analysis and the management information that it produces.

2. A secondary objective of this research was to define the relationships between efficiency, effectiveness, productivity, resource allocation, and capability.

3. Finally, in order to understand the types of managerial decisions which can be supported from the

application of the DEA/CFA models, thorough and clear explanations of these models were required and therefore provided by this research.

Research Questions

1. How do the DEA/CFA models define a relative frontier of efficiency for an individual unit being evaluated that takes into account all of the resources consumed and all of the outputs produced by that organization?

2. What type of management information can be obtained from the individual negative rates of substitution and positive rates of productivity that the models yield?

3. What are the limitations of the Data Envelopment Analysis and the Constrained Facet Analysis models, and under what conditions are these evaluation methodologies inappropriate for evaluating Air Force organizations?

4. How can network flow theory be applied in choosing an optimal mix of resources for each organization in the reference set, given budget constraints and organizational goals?

Scope

This research was limited to a brief review of the methods used in evaluating the technical efficiency of U. S. Air Force organizations including some of the limitations

associated with these evaluation methods. The available literature about the applications of Data Envelopment Analysis (DEA) and Constrained Facet Analysis (CFA) models in evaluating nonprofit organizations was also reviewed. Additionally, the literature review included discussions of network concepts and applications in both governmental settings and commercial settings which could be used in modeling and solving the resource allocation problem.

Finally, the author developed a network flow model of the resource allocation problem that can be used by managers as an aid to decision making. The model was based solely on the management information that is obtainable from application of the DEA and CFA efficiency measurement methodologies.

II. Literature Review

Overview

This literature review will define the efficiency, effectiveness, productivity, and capability aspects of organizational performance. An explanation of the interrelationships among these terms in a military capability context will be presented leading to a description of the resource allocation problem. Additionally, this review will discuss some of the traditional approaches taken in measuring efficiency and effectiveness in military organizations. Then, new models for organizational efficiency evaluation such as Data Envelopment Analysis (DEA) and Constrained Facet Analysis (CFA) will be reviewed, and their applications to real world problems will be discussed. Finally, this literature review will examine some network theory concepts which have been applied in solving specific resource allocation problems in government and industry.

Key Terms: Definitions and Discussion

The terms efficiency, effectiveness, productivity, and military capability are defined in a variety of ways throughout the literature.

To avoid confusion and to enhance the readability of this report, these terms will be defined and discussed so

that the similarities, distinctions, and relationships among these terms will be made clear to the reader. As an aid to this discussion, Figure 1 will be used to represent the production process of an Air Force combat wing referred to as Wing K. The production process of Wing K receives two inputs, manpower and materiel, and transforms these inputs into two outputs, sorties flown and mission capable aircraft.

Efficiency. Efficiency relates to how well the production process of Wing K uses the available inputs to produce outputs; i.e., it can be viewed as "a ratio of weighted outputs to weighted inputs ...relative to some maximum possibility [4:431]."

Absolute Efficiency. To be able to measure the absolute efficiency of Wing K, the maximum outputs possible given the level of inputs would have to be known. Absolute efficiency is defined as the ratio of the sum of the actual outputs to the sum of maximum outputs which can be produced from the given inputs in the same relative proportions. For example, for Wing K "in the same relative proportion" means:

$$\frac{\text{Actual sorties}}{\text{Maximum sorties}} = \frac{\text{Actual mission capable aircraft}}{\text{Maximum mission capable aircraft}}$$

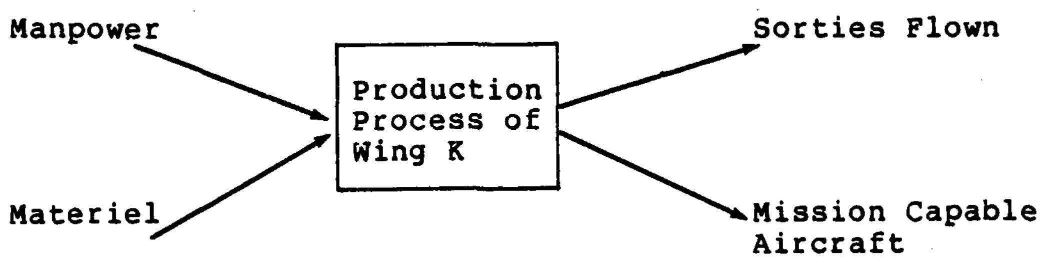


Figure 1.
Production Process of Wing K

Since the efficiency measure is a ratio, and the organization's actual outputs cannot exceed the maximum possible outputs, this measure of absolute efficiency produces a numerical value between zero and one [5]. The measurement of absolute efficiency of an organization is seldom possible because the maximum levels of output are usually unknown.

Absolute efficiency can be measured in mechanical systems. For example, in measuring the absolute efficiency of a power generator, the total mechanical energy supplied to the generator is known. Thus, it is possible to compute the ratio of actual electrical energy produced by the generator to the maximum amount of energy which could be produced if all mechanical energy could be transformed into useful electrical power. The theoretical maximum levels of outputs in complex organizations are generally not known, and some other measure of efficiency must be used [7].

Relative Efficiency. Farrell [10] proposed a solution to the problem of measuring the efficiency of organizations when the theoretical maximum output levels are not available. He conceptualized a way to measure efficiency in a relative sense by using the maximum observed output levels in lieu of theoretical maximums. In Farrell's words ". . . it is far better to compare performances with the best actually achieved than with some unattainable ideal" [10].

Therefore, relative efficiency is defined as the ratio of actual outputs to the highest outputs observed. For example, if Wing K flew 1,000 sorties and maintained an average of 16 mission capable aircraft during the month of June and if Wing J used the same amounts of resources but flew 1200 sorties and averaged 19.2 mission capable aircraft during the same time period, then, Wing K's efficiency relative to J would be:

$$
\begin{aligned}
(1000 + 16) / (1200 + 19.2) &= (1000 + 16) / 1.2\ (1000 + 16) \\
&= 1 / 1.2 \\
&= .833
\end{aligned}
$$

Farrell's idea was theoretically sound, but his method for computing the relative efficiency of a production process was unwieldy and impractical for large problems involving multiple inputs and multiple outputs. This computational difficulty was not satisfactorily resolved until Charnes and others [5] developed the Data Envelopment Analysis model.

<u>Frontier</u>. For the purposes of this research, a frontier is formed by those organizations in a reference set that are rated efficient by the DEA and CFA models. The frontier is used by the DEA and CFA models to evaluate an inefficient organization. It is the "yardstick" by which an organization's efficiency is measured. The term <u>Frontier</u>

Facet refers to the portion of the frontier used as the yardstick for a particular organization. This facet region is composed of efficient organizations which have input and output characteristics similar to the organization being evaluated.

Effectiveness. Effectiveness is defined as the ratio of actual output to planned output [7]. For this research, effectiveness refers to how well an organization is meeting its objectives or goals. The degree of effectiveness achieved by an organization does not reflect how efficiently resources were applied in obtaining the desired results. For example, if Wing K planned to fly 100 sorties during the month of June, but instead flew only 95, then Wing K is said to be 95% effective in meeting its sortie goal. It may be possible for Wing K to be 100% efficient in achieving that 95% effectiveness rating, but since Wing K did not meet or exceed its objective it must be classified as both efficient and ineffective. It is also possible for Wing K to be inefficient and ineffective, efficient and effective, or effective and inefficient.

Capability. For the purposes of this discussion, Clark's definition of military capability will be used [6:4]:

> . . . the maximum combat activity that one can reasonably expect to be produced by military units operating in a particular combat scenario given the available technology, the current levels of resources and the managerial abilities of commanders and supervisors.

Clark's definition of military capability implies that Wing K's capability is dependent on its state of readiness to deploy combat ready forces to carry out the tasks outlined in Wing K's operational plans. Intuitively, when the capability of an organization is inadequate, commmanders must identify the limitations or shortfalls so that corrections can be made through management actions or budget programs [6:4].

Productivity. Productivity is a combination of effectiveness and efficiency, and it is closely linked with the ability to carry out the assigned mission with the least amount of resources [7]. As noted by DODI 5010.34 [9]:

> The efficiency with which organizations utilize all types of fund resources (operating and investment) to accomplish their mission represents total resource productivity. The efficiency with which organizations utilize labor resources to accomplish their mission represents labor productivity. [underlining added] The primary objective of the DOD productivity program is to achieve optimum productivity growth (increase the amount of goods produced or services rendered in relation to the amount of resources expended) throughout the DOD.

Achieving Productivity Growth

The DODI 5010.34 definition of productivity implies that DOD managers should expect the organizations under their stewardship to be both efficient and effective and should allocate resources to achieve the greatest growth in military productivity and capability. Clearly, management must be able to measure efficiency, effectiveness, and

productivity before they can model the allocation process to achieve "optimum productivity growth" [9].

Traditional Approaches to Productivity Measurement and Evaluation. Sometimes productivity and efficiency are considered to be synonymous with profitability, which in commercial enterprises might be appropriate. The availability of a balance sheet and its corresponding "bottom line" makes money a convenient common denominator in commercial firms. But in not-for-profit organizations there is no balance sheet and no "bottom line" to provide the benchmark for evaluation. Simon states that in noncommercial firms the factors of production are not always measurable in monetary terms (e.g., public safety, or national defense), and ". . . monetary measure of outputs is usually meaningless or impossible [21:172]."

Comparing the performance of the nonprofit organizations and rating their efficiency have been recognized as difficult management problems [6]. Nonprofit organizations are difficult to compare because of the lack of a balance sheet (as explained by Simon), the multiplicity of outputs that an individual organization produces, and the many different types of the resources consumed by these organizations.

Historically, nonprofit organizations such as the U.S. Air Force have relied on partial measures of performance which typically appear in the form of ratios. For example,

the "sortie rate" of a tactical fighter wing (number of sorties flown in a month divided by the number of aircraft assigned) is one such measure. This performance measure has long been accepted by Air Force managers because it is easily computed and readily understood. Furthermore, this sortie rate ratio is a convenient way to measure a single valued output.

But, by definition, partial performance indicators such as ratios do not provide a single integrated measure of total performance which takes into account all outputs produced and all resources consumed. The lack of an integrated measure represents the most serious drawback in the ratio approach to performance evaluation. Managers who desire a comprehensive view of performance would need to analyze all ratios (all the different combinations of inputs to outputs) to form an opinion of organizational efficiency.

For example, if there were ten types of resources consumed and five outputs produced, then managers would need to examine 10 times 5, or 50, ratios. In practice, decision makers often focus on one or two ratios (for example, sortie rate and percent filled rate) and assess total organizational performance based on these partial measures.

Regression analysis is another method used in the evaluation of Air Force organizations. In regression analysis, all inputs are viewed as independent variables, while outputs are treated as dependent variables.

If an organization has only one type of output (e.g., combat sorties flown), this form of analysis and performance rating might be adequate. But if the number of outputs is greater than one (e.g., combat sorties and training sorties), then the interaction between the outputs is more difficult to model.

Simple linear regression cannot deal with situations where multiple outputs and multiple inputs must be considered simultaneously [14]. So, in practice, management rates the performance of an organization based on a combination of partial measures and a subjective assignment of weight factors for each of the measures [7]. As a result of partial measurement, scarce resources could be allocated to sustain inefficient operations while truly efficient organizations are denied the resources they need, which means that opportunities to increase productivity growth are lost.

New Models of Efficiency Measurement. The Data Envelopment Analysis (DEA) model developed by Charnes, Cooper, and Rhodes was a major breakthrough in performance evaluation. The DEA model is able to take into account simultaneously all of the inputs consumed and all of the outputs produced by each organization in a reference set and then assign an efficiency rating to each of the organizations without resorting to a priori assignment of weights to inputs and outputs. This model is based on a

concept developed by M. J. Farrell [10]; i.e., the efficiency ratings given by DEA are based on an observed frontier of productive efficiency, not an assumed or theoretical one [4,5]. Farrell's idea was to use an empirically defined efficiency frontier as the benchmark for evaluations. The DEA model is a deterministic, nonlinear, nonconvex programming model, with an ordinary linear programming equivalent. Upon solution, the DEA model yields a scalar measure of the efficiency of each participating unit. For a complete explanation of the model, the reader is referred to Appendix A.

Constrained Facet Analysis (CFA), a refinement of the DEA model, was developed by Clark [6] as part of his doctoral dissertation. The CFA model provided a solution to one major problem discovered during field applications of DEA. The DEA model was found to overestimate the efficiency of some organizations in the reference set.

The DEA and CFA models have the advantage of being able to rate the relative efficiency of an organization without requiring management to assign weights to the resources consumed and the outputs produced; i.e., the models will assign the weights in a way that will maximize the relative efficiency rating given. In addition to a relative efficiency rating, the models identify which efficient organizations were used as a standard for rating any particular organization. Finally, the manager will have

specific information indicating possible sources of inefficiency, and the manager will know which resources could be reduced and which outputs could be augmented (by specific amounts) to achieve an efficient rating for the organization.

The DEA model has been tested and validated at the University of Texas at Austin by the Education Productivity Council, a network of 25 school districts [4]. DEA was also applied in measuring the relative efficiency of courts [19], and of a complex hospital system [20].

At the Air Force Institute of Technology (AFIT), a 1984 graduate thesis examined the feasibility of applying CFA to measure the efficiency of an Air Logistics Center hydraulic shop [16]. The results of the AFIT thesis suggested that CFA was perceived by Air Force managers as an adequate model to use in measuring the efficiency of a depot level maintenance organization, and that it had potential for use in achieving productivity growth.

Bessent and others list the following warnings in using DEA or CFA [1,2]: The models are very sensitive to the accuracy and completeness of the data base, so great care must be exercised in the selection of inputs and outputs to be used in the evaluation to ensure that important inputs or outputs are not omitted. Failure to include important input and output measures could cause inaccurate efficiency ratings and distorted efficiency frontiers. Additionally,

the group of organizations to be evaluated must all produce the same type of outputs and must use some nonzero amount of all of the inputs.

Allocating Resources For Productivity Growth. The management information resulting from the application of the DEA and CFA models presents a new opportunity in determining resource allocations which achieve productivity growth. However, great care must be exercised to ensure that resource allocations or reallocations will result in increased effectiveness for the overall system as well as increased efficiency for each individual organization.

For example, management should not withdraw resources from organization K, an inefficient and ineffective unit, if the reallocation would drive it to be even more ineffective. The approach suggested by Clark [7] in dealing with the allocation problem could best be described by the matrix shown in Figure 2. Clearly, an organization should be in cell I; and if it is not, then it should be striving to reach cell I in order to achieve productivity growth. Clark suggests that the path that an organization should follow to achieve cell I will largely depend on where the organization starts in relation to effectiveness [7].

For example, if an organization is both inefficient and ineffective (cell IV), perhaps it should be allowed to retain its current levels of resources and should be

	Effective	Ineffective
Efficient	I	II
Inefficient	III	IV

Figure 2.
Organizational Effectiveness and Efficiency Matrix [7]

directed to increase its efficiency which in turn would increase outputs and effectiveness. So, an organization in cell IV should strive for cell I by way of cell II. An organization in cell II, ineffective but efficient, should strive for for cell I through output augmentation. An organization in cell III, effective but inefficient, should strive for cell I through resource conservation. Finally, an organization in cell I which is both effective and efficient should attempt to advance the frontier by seeking either resource conservation or output augmentation opportunities.

Given that effectiveness and efficiency can be measured, the question now becomes how should resources be allocated to achieve optimal productivity growth? Network theory has been used successfully by both industry and government in solving specific resource allocation problems. Network algorithms are available which can solve large scale mathematical formulations of complex resource allocation problems. One of the main reasons for the widespread use of networks in industry and government is that information can be presented in an easy to see, easy to understand, graphic format. Another major reason is that when dealing with large problems the savings in computation time accrued as a result of using network algorithms can be substantial [11:1215].

Network Flow Theory Applications

Barnes and Jensen [17] referred to some practical situations, such as distribution systems, which can be represented by networks because they have the characteristic of flow. Network flow models generally employ one of two types of networks: the pure network and the generalized network. A pure network flow model is distinguishable from a generalized network because flows through a pure network's arcs are not allowed to be modified, whereas flows through generalized network arcs may be increased or attenuated.

Glover and Klingman state that a "wide array of problems in production, distribution, financial planning, project selection, facilities location, resource management, and budget allocation fall naturally in the network domain [13:363]." Problems too large or too difficult to accommodate in 1970 can now be handled routinely by the new network computer codes and improved solution methodologies. Glover and Klingman cite the fact that a problem with 1000 nodes (equations) and 7000 arcs (flow paths between nodes) can be solved in about 8 seconds using advanced network code on an IBM 360. This same problem would take about 20 minutes using the same machine with the best commercial linear programming packages [13:364].

Pure Network Flow Applications. The applications cited by Glover and Klingman [13:364] for pure network modeling efforts include the following problems:

* shortest path
* assignment
* transportation
* transshipment

A transportation problem could be modeled using supplies, demands, and costs. In the example from Glover and Klingman (see Figure 3) A and B may be thought of as warehouses while nodes 1, 2, and 3 represent customers. The arcs indicate possible ways to ship the goods, while the boxes along the arcs represent the cost per unit. The absence of an arc from node B to node 2 indicates that warehouse B cannot ship to customer 2. In this simple example the objective is to minimize the total cost of shipping some commodity, perhaps tons of JP-4 fuel, from warehouses A and B to customers 1, 2, and 3.

Generalized Network Flow Applications. Generalized networks have arcs which increase or attenuate flow by a specified multiple [17]. Glover, Hultz, Klingman and Stutz describe generalized network problems as "a type of linear programming problem" [11:1209] and thus it can be solved by any linear programming package. However, Glover *et al* also point out that none of the available linear programming packages are capable of exploiting the structure of the network to computational advantage, an important attribute considering the fact that computer time is relatively expensive.

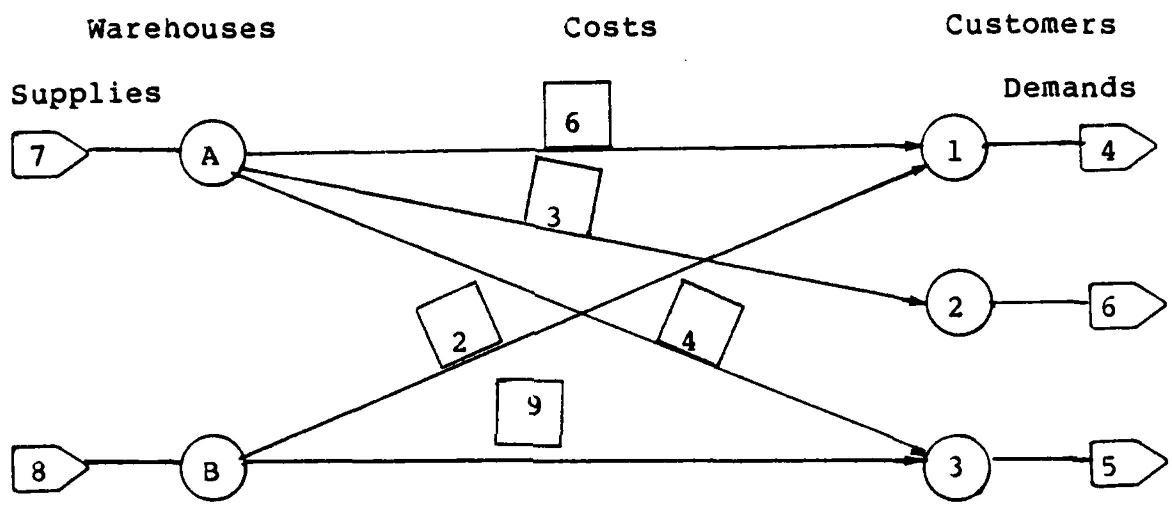


Figure 3.
Network Representation of the Distribution Problem [13:364]

There are two ways in which the arc multipliers for a generalized network function: a) they act to modify the amount of flow passing through the arc, or b) they transform the flow from one type of good to another. In the first case, the generalized network model can be used to model evaporation, seepage, machine efficiencies, etc. In the second case, the generalized network can be used to model the manufacturing process, production, crew scheduling, etc.

Network models can be constructed to specify integer flows. This attribute enables the modeling of situations where resources must be allocated in integer amounts (e.g., transfer 3 technicians, instead of 2.5). Integer flow is depicted in Figure 4 by adding an asterisk on the arc from node O to node A which implies that either the lower bound of zero or the upper bound of one will flow over the arc instead of fractions in between. The bounds are shown in a parenthesis over the arc as illustrated in Figure 3. The triangle in Figure 3 represents the arc multiplier; e.g., if one unit flows from node O along arc to node A, it will arrive at node A as three units which are subsequently distributed to nodes 1, 2, and 3.

These examples illustrate the versatility of networks in modeling resource utilization problems. Other areas of widespread application of network theory in the Air Force are Project Evaluation and Review Technique (PERT) and Critical Path Method (CPM).

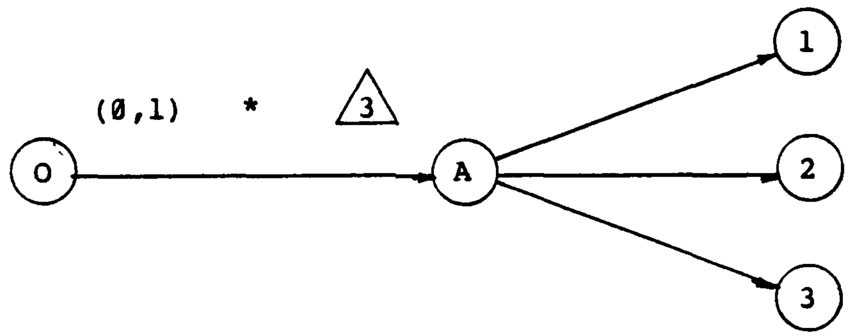


Figure 4
Generalized Network with Integer Flow Restrictions [11:1213]

Both techniques are now used in project planning, and their applicability has been demonstrated since the early 1960's.

Conclusion

The Department of Defense has recently come under increasing pressure to be more efficient and to exercise good stewardship in its expenditure of public funds. Additionally, there is a need within the Department of Defense to increase readiness and to be more productive. Ostensibly, methods that evaluate the efficiency and productivity of organizations in the DOD are of prime importance to the services and to commanders.

Evaluating the efficiency and productivity of nonprofit organizations, such as the DOD, has long been recognized as a difficult problem because these organizations are not judged by a balance sheet and its "bottom line." Instead, DOD produces a multitude of outputs that contribute to national defense.

The Data Envelopment Analysis and Constrained Facet Analysis models provide an approach to determining the overall relative efficiency of an organization when compared to others. These approaches overcome some of the difficulties previously encountered in using ratio analysis and regression analysis to evaluate performance.

This author is not suggesting that DEA and CFA are "panaceas" for evaluating the efficiency and productivity of nonprofit organizations. The models are very sensitive to data accuracy, so great care must be exercised in the selection and measurement of inputs and outputs to be used in the evaluation. Improper measures and incorrect data would cause inaccurate efficiency ratings and distorted efficiency frontiers. Furthermore, the group of organizations to be evaluated by DEA or CFA must all produce the same type of outputs and must use nonzero amounts of all of the inputs. And finally, great care must be exercised in evaluating an organization against itself over time using DEA or CFA because no satisfactory method has yet been developed to account for all of the time dependencies within the data.

The review of network theory presented in the preceding section described applications of pure and generalized network models. It is the intention of this author to use Constrained Facet Analysis to derive marginal rates of productivity and marginal rates of substitution in a group of related organizations, and then use these various rates in the formulation of a generalized network model which provides a solution for the resource allocation problem.

III. Methodology

Introduction

In the previous two chapters the discussion focused on some of the problems encountered when assessing the efficiency of military units and some of the obstacles to achieving productivity growth. Also discussed were two methodologies, Data Envelopment Analysis and Constrained Facet Analysis, which were recently developed to overcome many of the problems encountered in measuring productivity. Now the question becomes: given a budget, given a method to compute the apparent production efficiencies for each of the organizations under consideration, and given the desired goals of production for the group of organizations, how can resources be allocated among a group of organizations to best meet the desired goals?

This chapter provides a description of the network model used in this research to answer the question posed above. A test case was developed (with some simplifying assumptions) to show how resources under the control of a military headquarters might be allocated to a group of subordinate tactical Air Force organizations in order to achieve productivity growth.

Suppositions and Assumptions For the Test Case

Suppose there are twelve tactical fighter wings under the control of a single parent headquarters, perhaps a Numbered Air Force; and suppose the headquarters desires to increase the combined efficiency and effectiveness of the group in order to increase overall capability. Furthermore, suppose each wing uses two resources (manpower and materiel) and produces two outputs (sorties and mission capable aircraft). The two-input, two-output case was selected to simplify modeling and analysis yet provide sufficient complexity to illustrate the important features of the resource allocation problem.

Assume also that:

1. Some portions of the resources used by organizations are transferable to other organizations at no cost, if such transfer will increase the aggregate output (capability) of the group.

2. All twelve wings possess the same type of weapon system.

Finally, the following assumptions which were first reported in Bessent, Bessent, and Clark [2] were made to satisfy the requirements for the application of the Constrained Facet Analysis efficiency evaluation methodology:

1) Outputs [selected for inclusion in the analysis] represent important unit goals.

2) All measures [of input] are appropriate and exist in nonzero amounts.

3) [The input measures selected for inclusion in the analysis] represent all the physical quantities used by the units towards attainment of outputs.

4) There is a conceptual basis for believing that changes in the outputs should be caused by changes in the inputs.

5) The magnitude of physical input and output quantities are [taken into account by the measures chosen].

6) [The chosen measures take into account] the quality of inputs and outputs.

Data Generated for the Test Case

The data used for the test case was a contrived data set adapted from Clark's doctoral dissertation [6]. Clark's data set consisted of 14 organizations, each consuming 4 inputs and producing 3 outputs. It was used to generate a reduced set of experimental data for twelve Air Force organizations each consuming two types of resources and producing two types of outputs. The data generated for the test case are shown in Table I.

Table I

Test Case, Output and Input Data

	Outputs		Inputs	
	Y_1	Y_2	X_1	X_2
Wings	Number Of Sorties Flown	Mission Capable Aircraft Days	Manpower (thousand hours)	Materiel
A	15192	15794	1980	27026.031
B	10435	10083	1408	18354.098
C	13991	14552	1936	26906.664
D	12348	13771	1496	18479.665
E	17193	21667	2508	33367.097
F	9741	12795	1320	19187.467
G	12579	16848	1302	25029.964
H	6673	10178	924	12394.665
I	16010	16196	1980	26628.331
J	19661	22297	1980	36785.330
K	4640	4562	2640	8958.566
L	7532	10817	1188	27831.964
Total	145995	169560	20662	280949.842

Inputs and Outputs.

Output One (Y_1) represents the total number of sorties flown in a year by a wing. Clark [6] defines a sortie as the take-off, flight, and full stop landing (not touch-and-go) of one aircraft. According to Clark [6] sorties flown are valued output because they represent air crew training and the exercising of ground support functions to maintain high levels of personnel readiness as well as to keep mission essential equipment in good operating condition.

Output two (Y_2) represents mission capable aircraft days accumulated during the year. Output two is also a valued output because each wing is expected to maximize the number of mission capable aircraft available at any point in time in order to remain prepared for war. This output is computed by calculating the number of days in a year that each possessed aircraft is mission capable.

Input one (X_1) represents labor hours (in thousands of hours). Labor hours measure the size of the available work force and vary proportionately with the levels of flying and ground support activities at each wing.

Input two (X_2) represents a materiel index. This input takes into account a combination of factors including available aircraft, supply support, and mission essential equipment availability.

Test Case Procedure

Once the data set was developed, the organizations were evaluated to determine if any inefficiencies existed. The evaluation provided efficiency ratings and marginal rates of substitution and productivity for each of the organizations in the test case.

The frontier facet for each unit was identified and the marginal rates of substitution and productivity in the facet were listed. An explanation of how to derive the marginal rates of substitution and productivity is given in Chapter IV using specific values derived from the application of Constrained Facet Analysis [1] to the organizations in the test case.

At this point in the analysis two pieces of information required by the resource allocation methodology are known: a) the level of resources available, and b) the rates of conversion of inputs to outputs.

The third and final piece of information that the resource allocation methodology requires is the desired production goals set by management. These goals were arbitrarily set by the author at amounts corresponding to a 5% increase in the total level of each output for both output categories (Sorties and Mission Capable Aircraft Days). Then the resource allocation model presented in Figure 5 and discussed in the next section was formulated and solved, and the results were recorded.

Graphical Representation of the Resource Allocation Model. The resource allocation model shown in figure 5 is a generalized network with two source nodes, X1 and X2, representing the total resources available, and two sink nodes, Y1 and Y2, representing the total output amounts produced. Each individual organization was represented by a collection of four nodes, two nodes for the inputs and two nodes for the outputs. For example, in Figure 5, Wing A receives inputs from sources X1 and X2 at nodes X1A and X2A respectively; then the production process of Wing A (represented by the four arcs going from nodes X1A and X2A to nodes Y1A and Y2A) converts the inputs into outputs. The multipliers along the arcs of the production process equal the marginal rates of productivity of each of the inputs with respect to each of the outputs. The arc multiplier values used to model each production process are derived from the Constrained Facet Analysis efficiency evaluation methodology.

The limits on the maximum and minimum amounts of flow permitted on each of the input (output) arcs of an organization was set equal to the maximum and minimum amounts of input (output) that were observed in the facet for that unit. The limits on the amount of flow were set at previously observed amounts to guarantee that a feasible solution could be found. Some other feasible combination of bounds could have been chosen if management desired to specify the range of amounts to be transferred.

X_1 = Total amount of input 1 available

X_2 = total amount of input 2 available

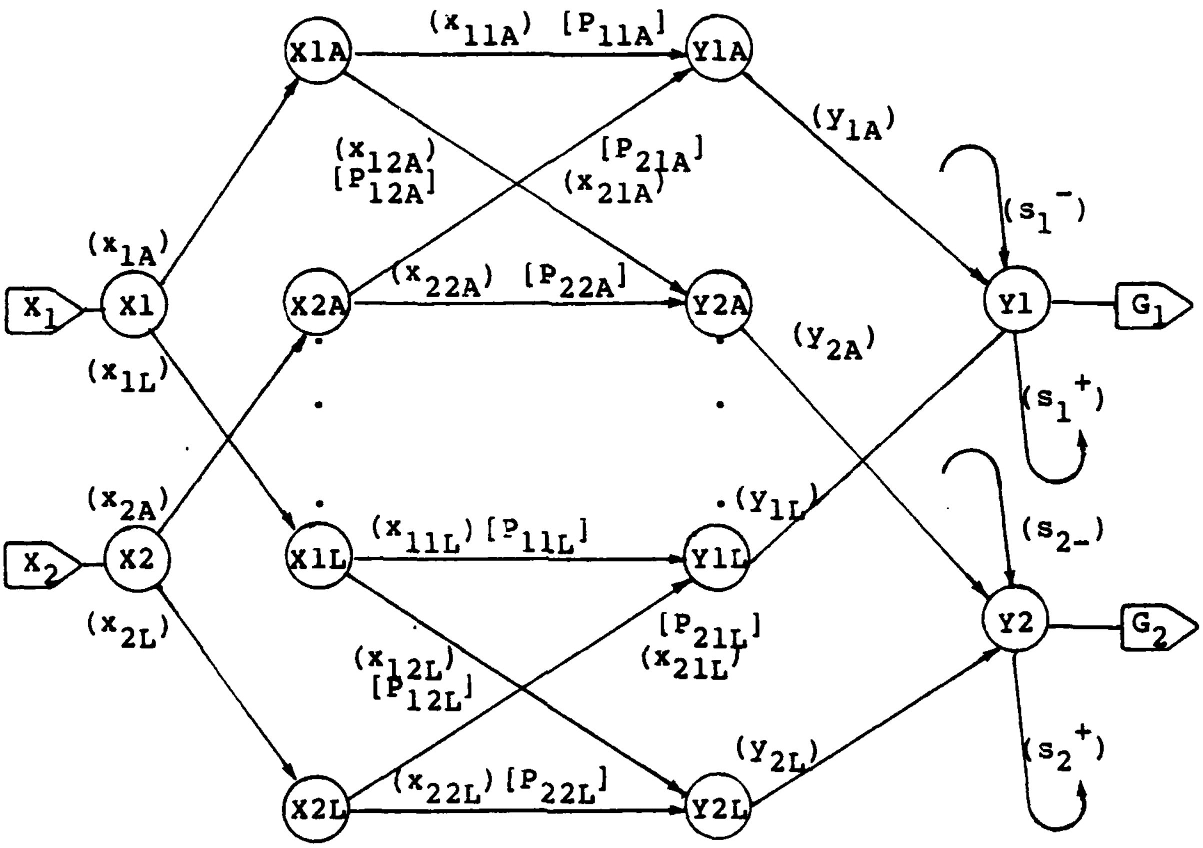
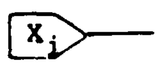


Figure 5.
Resource Allocation Network Model

Graphing Notation:

( ) - Flows

[ ] - Multiplier

 - Supply

 - Demand

The objective of the model was to minimize the output shortage at each of the sink nodes (see Figure 6) given the level of resources available, given the apparent rates of productivity of each resource with respect to each output, and given the desired production goal. The shortage from the goal is represented on the network as a negative surplus arc. Upon solution, the minimization routine seeks to find collective output amounts for Y_1 and Y_2 which meet or exceed the desired goals G_1 and G_2. Additionally, the resource allocation model will yield the identity of those organizations that were selected to give up resources as well as those organizations that were selected to receive them.

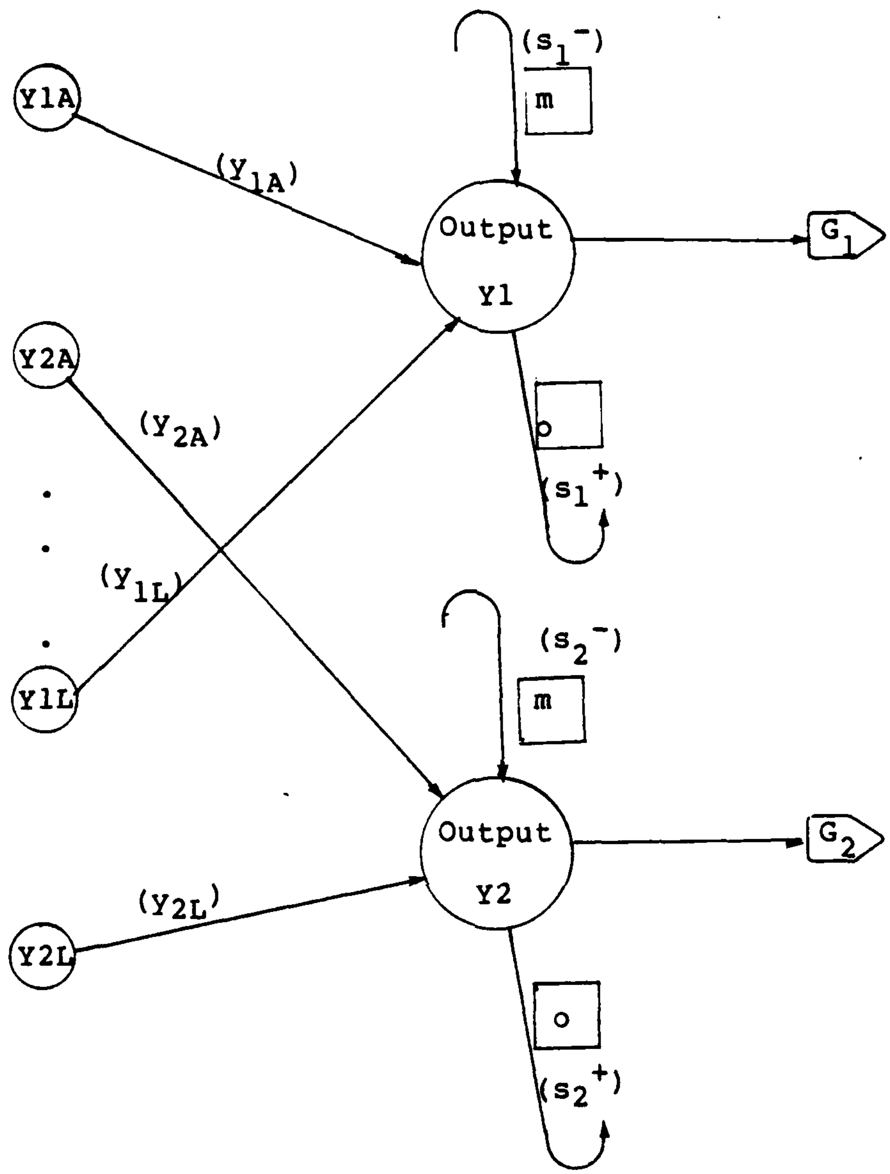


Figure 6.
Goal of Achieving a Total Amount of Output 1 and Output 2 Greater Than or Equal to Some Desirable Amounts G1 and G2 (Minimize $s_1^- + s_2^-$)

Graphing Conventions [11]:

(y_{rj}) - Flow

$\boxed{m}$ - Cost

(min, max) - Flow Bounds

G_r - Demand

Mathematical Formulation of the Generalized Network Resource Allocation Model. In the previous section, a graphic representation for the two-output, two-input resource allocation model was discussed. This section contains the mathematical formulation of the model in generalized network form which assumes the existence of multiple inputs and multiple outputs.

Let:

X_i = the total amount of resource type $i = 1, 2, \ldots, m$ which is available for allocation to organizations $j = 1,2,\ldots,n$

x_{ij} = the portion of X_i allocated to organization j for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ (variable computed by the model)

x_{irj} = the portion of x_{ij} allocated by organization j to the production of output y_r for $r = 1, 2, \ldots, s$ (variable computed by the model)

Y_r = the sum total amount of output type r produced by organizations $j = 1, 2, \ldots, n$ (variable computed by the model)

y_{rj} = amount of output y_r produced by organization j for $r = 1, 2, \ldots, s$ and $j = 1, 2, \ldots, n$ (variable computed by the model)

G_r = the goal established for y_r by management for $r = 1, 2, \ldots, s$

W_r = relative weight assigned by management to the achievement of goal G_r for $r = 1, 2, \ldots, s$

s_r^- = the amount that output y_r falls short of meeting goal G_r for $r = 1, 2, \ldots, s$ (variable computed by the model)

s_r^+ = the amount that output y_r exceeds goal G_r for $r = 1, 2, \ldots, s$ (variable computed by the model)

P_{irj} = is the apparent rate of productivity in the frontier neighborhood of organization j which specifies the rate at which input x_{irj} is converted into output y_r for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ and $r = 1, 2, \ldots, s$

The resource allocation model is formulated as follows:

Minimize $$\sum_{r=1}^{s} W_r s_r^- \qquad (3.1)$$

Subject To:

$$\sum_{j=1}^{n} x_{ij} = X_i \qquad (3.2)$$

;for $i = 1, 2, \ldots, m$

$$- x_{ij} + \sum_{r=1}^{s} x_{irj} = 0 \qquad (3.3)$$

;for $i = 1, 2, \ldots, m$
and $j = 1, 2, \ldots, n$

$$- \sum_{i=1}^{m} P_{irj} x_{irj} + y_{rj} = 0 \qquad (3.4)$$

;for $r = 1, 2, \ldots, s$
and $j = 1, 2, \ldots, n$

$$- \sum_{j=1}^{n} y_{rj} + y_r = 0 \qquad (3.5)$$

;for $r = 1, 2, \ldots, s$

$$y_r - s_r^+ + s_r^- = G_r \qquad (3.6)$$

;for $r = 1, 2, \ldots, s$

$$x_{ij},\ x_{irj},\ y_{rj},\ y_r,\ s_r^+,\ s_r^- \geq 0 \qquad (3.7)$$

;for $i = 1, 2, \ldots, m$
and $j = 1, 2, \ldots, n$
and $r = 1, 2, \ldots, s$

Output Bounds:

$y_{rj} \leq$ maximum output observed in the neighborhood (frontier facet) of unit j, or some higher amount determined by management

$y_{rj} \geq$ minimum output observed in the neighborhood (frontier facet) of unit j, or some lower amount determined by management

Input Bounds:

$x_{ij} \leq$ maximum input observed in the neighborhood (frontier facet) of unit j, or some higher amount determined by management

$x_{ij} \geq$ minimum input observed in the neighborhood (frontier facet) of unit j, or some lower amount determined by management

Expression (3.1) is called the objective function. This expression explicitly states that the objective of the mathematical program is to minimize the weighted sum of the amounts that outputs fall short of reaching the desired goals. The shortages are graphically shown in the test case example as slack arcs s_1^- and s_2^- (see Figures 5 and 6) and they are shown in the more general mathematical expression (3.6) as variables s_r^- for $r = 1, 2, \ldots, s$.

Equality (3.2) is the constraint which requires that the total amount of input i available for allocation be

allocated to organizations $j = 1, 2, \ldots, n$. In other words, the model may not allocate any more (or less) resources than available.

Equality (3.3) is another conservation of flow constraint and explicitly requires that each organization $j = 1, 2, \ldots, n$ use all of the allocated resource x_{ij} in the production of its different outputs y_r.

Equality (3.4), still another conservation of flow constraint, captures the production process of each organization $j = 1, 2, \ldots, n$ by using the different rates of productivity P_{irj} as a multiplier to specify the rate at which each input $i = 1, 2, \ldots, m$ is converted into each output $r = 1, 2, \ldots, s$. Since each production process has different rates of conversion of inputs to outputs, the model must search for the most efficient allocation of resource x_{ij} among the different production processes subject to the minimum and maximum amounts of flow (bounds) allowed.

Equality (3.5) is the conservation of flow constraint for the sink nodes. This equality simply states that the total amount of output y_r for $r = 1, 2, \ldots, s$ is equal to the sum of the outputs y_{rj} from the different production processes $j = 1, 2, \ldots, n$.

Equality (3.6) compares each of the collective outputs y_r for $r = 1, 2, \ldots, s$ to a desired goal G_r for $r = 1, 2, \ldots, s$ through the use of slack and surplus

variables. A negative slack variable s_r^- allows flow into the network to augment flow output y_r when the flow of output y_r falls short of the goal G_r. A positive surplus variable s_r^+ allows flow out of the network when flow y_r exceeds the goal G_r.

Inequalities (3.7) are the nonnegativity restrictions on the variables that are to be computed by the model.

<u>Evaluation of Test Case Results and Allocation Model Validation</u>. Once the new allocations were determined, follow-on Constrained Facet Analysis was performed to verify that the new allocations resulted in a higher efficiency rating for each of the units. The CFA test involved using the 12 original organizations and their observed inputs and outputs as a reference set to evaluate each new combination of inputs and outputs resulting from the reallocation. Then comparisons of efficiency ratings were made to determine if the proposed allocations resulted in productivity growth as reflected by higher efficiency ratings.

Computer Resources Required

The computer code to be used for the DEA/CFA model is now available on the Burroughs B-29 system written in the Basic programming language. As an alternative, The Multi-Purpose Optimization System (MPOS) is available on the CDC CYBER computer system. The mathematical program for the generalized network used in the test case was coded for execution in the MPOS utility (see Appendixes D and E).

Summary of Methodology

This chapter described the data set used in finding an optimal allocation of resources among a group of twelve Air Force organizations under the control of a single headquarters. The hypothetical wings used two inputs, labor and materiel, and produced two outputs, sorties and mission capable aircraft. A description was given of the method of evaluation used for finding the efficiency ratings and the rates of productivity of each organization relative to the other organizations in the group.

Then, a two-input, two-output generalized network model was used to determine an optimal reallocation. A network model was chosen because it can be depicted graphically, can be easily translated into a linear programming model, and can be formulated to represent the structure of the production process. The mathematical model of the resource allocation network was also provided.

In the next chapter, the resource allocation methodology discussed above will be applied to the two-input, two-output test case.

IV. Findings and Analysis

Introduction

This chapter reports the results of applying the resource allocation methodology to the two-input, two-output test case discussed in Chapter III. Recall that the resource allocation methodology consisted of 1) efficiency evaluation, 2) identification of frontier facets and derivation of the marginal rates of substitution and marginal rates of productivity, and 3) an application of the resource allocation model given the level of resources available and the specific goals set by management.

Data Set Revisited

The data set presented in Table I in Chapter III consisted of input and output amounts for twelve fictitious Air Force tactical fighter wings, each utilizing some amount of resources X_1 and X_2, Manpower and Materiel, and producing some amount of outputs Y_1 and Y_2, Sorties and Mission Capable Aircraft Days. Table II is a reproduction of the data set previously presented in Table I. It was reproduced here so the reader could refer to it during the discussion that follows.

Table II

CFA Outputs and Input Data

	Outputs		Inputs	
Wings	Y_1 Number Of Sorties Flown	Y_2 Mission Capable Aircraft Days	X_1 Manpower (thousand hours)	X_2 Materiel
A	15192	15794	1980	27026.031
B	10435	10083	1408	18354.098
C	13991	14552	1936	26906.664
D	12348	13771	1496	18479.665
E	17193	21667	2508	33367.097
F	9741	12795	1320	19187.467
G	12579	16848	1302	25029.964
H	6673	10178	924	12394.665
I	16010	16196	1980	26628.331
J	19661	22297	1980	36785.330
K	4640	4562	2640	8958.566
L	7532	10817	1188	27831.964
Total	145995	169560	20662	280949.840

Efficiency Evaluation and Derivation of the Rates of Substitution and Productivity

Constrained Facet Analysis was performed for all of the wings in the reference set (see Table III). The efficiency information required by the resource allocation procedure was obtained from this analysis. The difference between the upper bound (DEA) and lower bound CFA relative efficiency ratings reveals the degree of nonenvelopment of an inefficient organization; i.e., the closer the two values are to each other, the closer the inefficient wing is to being enveloped by a frontier facet of efficient wings. The information gained from the comparison of the DEA and CFA values is valuable because it reveals to management just how much of an "outlier" the inefficient organization is. For the purposes of this research, the items of interest from the CFA evaluation are the efficiency ratings and the apparent marginal rates of productivity observed in the frontier facet for each wing.

Table III summarizes the efficiency ratings given by the CFA model. A rating of 1.0 signifies that the model considers the organization to be 100% efficient relative to all the other organizations in the reference set. The "Units in the Facet" columns of Table III list those efficient wings that the model found in the neighborhood of the wing being evaluated. Recall from the discussion of the DEA model that the upper bound efficiency value is

Table III

Efficiency Rating and Wings in the Facet for Each of the Aircraft Wings Being Evaluated By DEA and CFA

Wings	DEA Upper Bound Efficiency Rating	Units in the Facet	CFA Lower Bound Efficiency Rating	Units in the Facet
A	0.891859	D J	0.875737	D G J
B	0.878334	D J	0.848324	D G J
C	0.833538	D J	0.818406	D G J
D	1.000000	D H	1.000000	D G H
E	0.852942	D G H	0.852942	D G H
F	0.894467	D G H	0.894467	D G H
G	1.000000	D G H	1.000000	D G H
H	1.000000	H	1.000000	D G H
I	0.945825	D J	0.922893	D G J
J	1.000000	D G J	1.000000	D G J
K	0.77513	D	0.294928	D G J
L	0.703644	G	0.599876	D G H

obtained when the multipliers (or weights) for the inputs and outputs are assigned by the model in order to achieve the highest possible efficiency rating, and multipliers might be given a value of zero. On the other hand, to obtain the lower bound efficiency value CFA tries to force all multipliers to take a value greater than zero by bringing other nearby frontier organizations into the evaluation. For example, Wing A was given an upper bound efficiency rating of .891599 when it was compared to Wings D and J; while a lower bound of .875737 was obtained by adding Wing G to the facet of units used as a reference in evaluating Wing A.

The multipliers listed in Table IV were assigned by the Constrained Facet Analysis model. The u_i^* (for i=1,2) represent the multipliers (or weights) assigned by the CFA model to the Y_1 and Y_2 output observations of each wing, and the v_i^* (for i=1,2) represent the multipliers assigned by the CFA model to the X_1 and X_2 input observations of each wing. These multipliers were assigned to the input and output observations of the unit being rated in order to achieve the highest possible relative efficiency rating given that the lower bound facet was used as a reference. The asterisk (*) is the notation convention used by the developers of the model to indicate the optimal model solution, and therefore the <u>optimal</u> multiplier assignment.

Table IV

CFA Multipliers Assigned by the Model to Inputs and Outputs

	Output Multipliers		Input Multipliers	
	Y_1	Y_2	X_1	X_2
Wings	Number Of Sorties Flown	Mission Capable Aircraft Days	Manpower (thousand hours)	Materiel
	u_1^*	u_2^*	v_1^*	v_2^*
A	.0000439	.0000132	.0002842	.0000162
B	.0000629	.0000190	.0004076	.0000232
C	.0000445	.0000134	.0002883	.0000164
D	.0000380	.0000385	.0003136	.0000287
E	.0000218	.0000221	.0001797	.0000165
F	.0000394	.0000399	.0003249	.0000298
G	.0000337	.0000342	.0002782	.0000255
H	.0000589	.0000597	.0004856	.0000445
I	.0000442	.0000133	.0002860	.0000163
J	.0000379	.0000114	.0002454	.0000140
K	.0000490	.0000148	.0003174	.0000181
L	.0000324	.0000329	.0002676	.0000245

The following computations for efficient Wing J demonstrate how the observed input and output amounts can be used to determine the efficiency rating of 1.0:

$$\frac{(Y_1)\ (u_1^*) + (Y_2)\ (u_2^*)}{(X_1)\ (v_1^*) + (X_2)\ (v_2^*)} =$$

$$\frac{(19661)\ (.0000379) + (22297)\ (.0000114)}{(1980)\ (.0002454) + (36785.33)\ (.0000140)} = 1.0$$

The negative rates of substitution and the positive rates of productivity can be computed from the multiplier values provided by the CFA model. From the DEA and CFA literature [6:173] we know that the facet of wing K is defined so that the set of all vectors (y_1, y_2, x_1, x_2) in the facet satisfy the following equality:

$$u_{1K}^* y_1 + u_{2K}^* y_2 - v_{1K}^* x_1 - v_{2K}^* x_2 = 0 \qquad (4.1)$$

where y_1, y_2, x_1, x_2 vary within the facet and u_{rK}^* and v_{iK}^* for $i = 1, 2$ and $r = 1, 2$ are treated as constants.

The rates of substitution between inputs (outputs) is obtained by taking the partial derivative of each of the inputs (outputs) with respect to each of the other inputs (outputs).

For example, in the two-input, two-output case, if output 1 of Eq. (4.1) is isolated as shown below in equality (4.2) and then if the partial derivative of output 1 with respect to output 2 is taken as shown below in (4.3):

$$y_1 u_{1K}^* = x_1 v_{1K}^* + x_2 v_{2K}^* - y_2 u_{2K}^* \tag{4.2}$$

$$\frac{\partial (y_1 u_{1K}^*)}{\partial y_2} = \frac{\partial}{\partial y_2}(x_1 v_{1K}^* + x_2 v_{2K}^* - y_2 u_{2K}^*) \tag{4.3}$$

the following equality results:

$$\frac{\partial y_1 \; (u_{1K}^*)}{\partial y_2} = 0 + 0 - u_{2K}^* \tag{4.4}$$

Which implies that:

$$\frac{\partial y_1}{\partial y_2} = - u_{2K}^* / u_{1K}^* \tag{4.5}$$

Substituting the u_{rK}^* values (for r= 1,2) found in Table IV for the multipliers in equation (4.5):

$$\frac{\partial y_1}{\partial y_2} = - .0000148/.0000490 = - .3020$$

Which means that if Wing K were efficient it should expect to give up approximately .3020 mission capable

aircraft days for every extra sortie that it flies if all inputs remain constant at the current level.

The same type of negative substitution relationship that exists on the output side of equality (4.1) holds for the input side.

The rates of productivity are derived in a similar manner. For example, in the production process of Wing K (equality 4.1), the partial derivative of $(y_1 u_{1K}^*)$ with respect to (x_1) will result in the following:

$$\frac{\partial(y_1 u_{1K}^*)}{\partial(x_1)} = v_{1K}^* \tag{4.6}$$

which implies that:

$$\frac{\partial y_1}{\partial x_1} = \frac{v_{1K}^*}{u_{1K}^*} \tag{4.7}$$

Substituting the multiplier values found in Table IV into equality (4.7) will yield the following marginal rate of change in output y_1 which would result from one additional unit of input x_1:

$$\frac{\partial y_1}{\partial x_1} = \frac{.0003174}{.0000490} = 6.4776$$

A complete listing of the marginal rates of substitution and productivity for Wing K can be found in Table V. The number shown in each column i and row j is the partial derivative of the column variable with respect to the row variable; e.g., the number 6.4776 = P_{11K} when the column variable is input i =1 and the row variable is output r = 1. All constant amounts P_{irj} must be supplied to the allocation model. These constants, which serve as multipliers on input arcs, convert input flows into output flows.

The number in parenthesis under each input and output variable in Table V is the appropriate CFA multiplier value from row K Table IV, and the numbers in the matrix of Table V are the partial derivatives of each variable in the column with respect to the row variable.

Table V

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing K

	1. ∂y_{1K} (.0000490)	2. ∂y_{2K} (.0000148)	3. ∂x_{1K} (.0003174)	4. ∂x_{2K} (.0000181)
1. ∂y_{1K} (.0000490)		-3.3108	.1544	2.7072
2. ∂y_{2K} (.0000148)	-.3020		.0466	.8177
3. ∂x_{1K} (.0003174)	6.4776	21.4459		-17.5359
4. ∂x_{2K} (.0000181)	.3694	1.2230	-.0570	

The negative numbers represent rates of substitution and the positive numbers represent rates of productivity in the proper facet of Wing K. For example, the number 6.4776 in the matrix for Table V, column 1 row 3, is the P_{11K} positive marginal rate of productivity which converts input 1 into output 1.

For a more rigorous explanation of the relationship of CFA multipliers to the marginal rates of substitution and marginal rates of productivity, the reader is directed to Clark [6:161]. The rates of substitution and productivity for all of the organizations in this test case can be found in Appendix B.

The Production Process of Wing K Revisited

The significance of the P_{irj} multipliers to this research is best illustrated by explaining the network in Figure 7. Suppose Wing K receives one unit of input 1 at node X1K. Node X1K in turn can flow some or all of input 1 across the arc from from node X1K to node Y1K or node Y2K. Nodes Y1K and Y2K collect flow output 1 and flow output 2 respectively.

When one unit of input 1, x_{1K}, departs node X1K and flows toward Y1K it will be converted into 6.4776 units of output 1, Y_{1k}, by the arc multiplier P_{11k}. If the unit of input 1, x_{1k}, flows toward node Y2K instead, it will be converted into 21.4459 units of output 2 by the arc multiplier P_{12K}.

The arc multipliers P_{irj} explicitly model the various production processes using observed, not theoretical, rates of productivity.

Goal Setting and Resource Allocation

The efficiency (and the various rates of productivity) of all organizations in the test case were determined (see Tables III, IV, V, and Appendix B).

The next step required in the research methodology was to explicitly state the goals being sought for each of the outputs. The goals were graphically presented as sink nodes in the test case network.

Management's preference for a particular goal can be reflected in the resource allocation model by assigning various weights (or costs) to the output shortfall arcs s_1^- and s_2^- in Figure 6.

The model was run for the test case by setting the desired output goals G_1 and G_2 to an amount that represented a 5% increase over the current level of collective output for Y_1 and Y_2; specifically, $G_1 = 153294.75$ and $G_2 = 178038$.

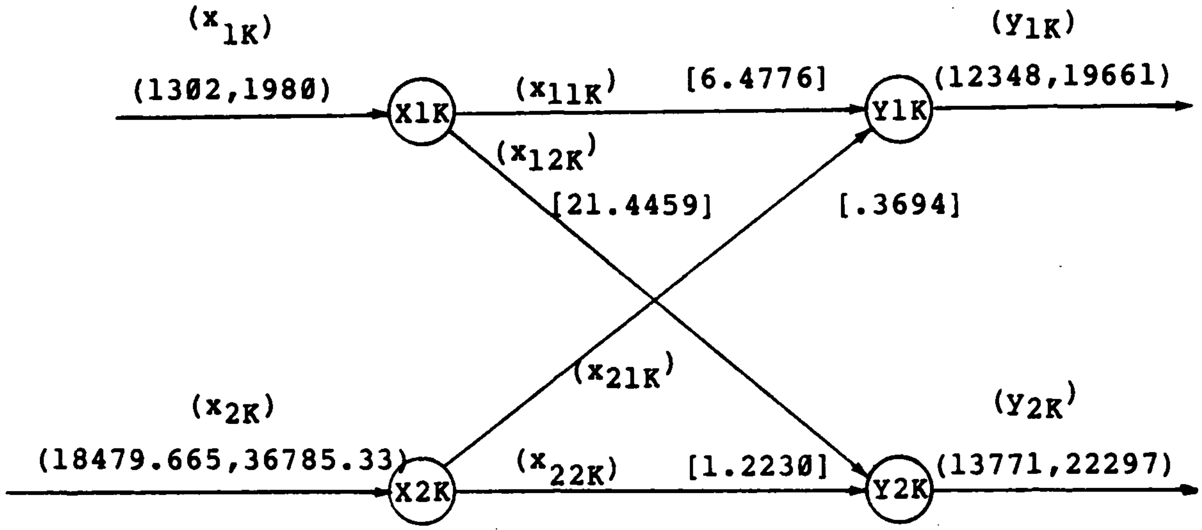


Figure 7.
The Production Process of Wing K

Arc Flow Notation:

( ) - Flow

(min, max) - Flow Bounds

[P_{irj}] - Multiplier

Limiting the Amount of Inputs Allowed to be Transferred by Bounding the Flow on Arcs

If the model was tried as suggested in Figure 7 with no bounds on the input flow, the model would favor the flows with the greatest conversion factors and would avoid sending flow through arcs in the production process which have low conversion factors. Therefore, bounding the flows was necessary to keep the model from selecting only one type of resource; e.g., it would be unrealistic to expect any wing to produce sorties and mission ready aircraft days from manpower alone without having materiel inputs as well (aircraft, supply support, and mission essential equipment). Clearly, any wing that expects to accomplish its mission must possess all required inputs.

The bounding method selected for this research consisted of setting the lower and upper bounds of all input and output flow equal to the minimum and maximum amounts of each type input and output observed from efficient wings which determine the facet for the production process.

The author concluded that this bounding scheme would be realistic since performance within the specified bounds of inputs and outputs had already been demonstrated by the wings in the evaluation. Other bounds could be substituted if these values were not considered to be feasible for the process being modeled.

A complete list of the network representation of all the production processes with corresponding bounds can be found in Appendix C.

<u>Results of Experimentation With Resource Allocation Model</u>

There were four basic trials performed with the test case data. A complete listing of the computer program used for resource allocation trials 1, 2, and 3 can be found in Appendix D. The first trial assumed equal preference for both goals; meaning that the weights W_1 and W_2 for goal shortages s_1^- and s_2^- were both set equal to one. The second trial was performed indicating a preference for the first goal G_1 by setting W_1 equal to one and W_2 equal to zero.

The third trial was made indicating a preference for the second goal G_2 by setting W_2 equal to one and W_1 equal to zero.

Finally, a fourth trial was made by modifying the objective function so that the model would maximize $y_1 + y_2$ given the same set of constraints as before. The computer code for the fourth trial can be found in Appendix E.

The model performed successfully all four times implying that productivity growth was possible if the current level of resources were reallocated to achieve greater efficiency and effectiveness.

The allocations for trials 1, 2, and 3 were the same and are shown in Table VI. Table VII lists the proposed allocation when the objective is to maximize both outputs y_1 and y_2. Tables VI and VII can now be contrasted with Table II to compare the proposed allocation of resources to the amounts originally observed.

When the model was asked to allocate resources to meet or exceed G_1 or G_2 or both, it exceeded both goals G_1, Sorties, and G_2, Mission Capable Aircraft Days, by 27624 sorties and 2246 mission capable aircraft days respectively.

The experimental results of trial 4 are shown in Table VII. In trial 4, the model generated an allocation of resources which produced outputs of 164,298 sorties and 234,870 mission capable aircraft days. These levels of output represent a 12.5% and 38.5% increase respectively over current levels of output.

In all cases, the model generated allocations and production levels that were within the range of observed, not theoretical, allocations and production levels. Thus, generation of feasible allocations and production levels was accomplished by restricting the input and output flows into each production process to the minimum and maximum levels of input and output observed at the frontier.

Table VI

New Allocation of Resources For Trials 1, 2, and 3

	Outputs		Inputs	
Wings	Y_1 Number Of Sorties Flown	Y_2 Mission Capable Aircraft Days	X_1 Manpower (thousand hours)	X_2 Materiel
A	19661	13771	1980	29764.945
B	15486	13771	1980	18479.665
C	19661	13771	1980	29794.949
D	12579	16848	1496	22908.230
E	12579	16848	1496	22892.510
F	12579	10178	1648	22878.121
G	12579	16848	1496	22898.695
H	12579	16848	1496	22297.220
I	15483	13771	1980	18479.665
J	19661	13771	1786	33132.082
K	15492	13771	1980	18479.665
L	12579	13418	1496	18314.052
Total	180918	180284	20662	280949.800

Table VII

New Allocation of Resources Given
An Objective of Maximizing Both Outputs

	Outputs		Inputs	
Wings	Y_1 Number Of Sorties Flown	Y_2 Mission Capable Aircraft Days	X_1 Manpower (thousand hours)	X_2 Materiel
A	12933	13771	1980	24093.443
B	12912	18015	1980	29135.877
C	12924	13771	1786	23997.758
D	12579	16848	1496	21922.886
E	12579	16848	1496	21962.276
F	12579	16848	1496	22033.177
G	12579	16848	1496	21942.808
H	12579	16848	1496	21924.667
I	12918	13771	1980	24013.828
J	17476	13771	1980	24061.145
K	19661	13771	1980	23990.699
L	12579	16848	1496	21921.235
Total	164298	234870	20662	280949.800

Resource Allocation Model Validation

To validate the resource allocation model, Constrained Facet Analysis was performed for each organization which received new levels of outputs and inputs from the allocation. The organizations comprising the reference set for the CFA evaluations were the 12 organizations in the original test case with their observed levels of production (see Table II). Each proposed organization with its levels of input and output specified by the resource allocation model was analyzed relative to the original reference set of 12 wings (see Table VI).

The results of the Constrained Facet Analysis evaluation are summarized in Table VIII. All of the organizations generated by the resource allocation model were as efficient or more efficient than their counterparts were at the original levels of production and consumption.

Another observable result of the CFA evaluation was that the input and output mixes of the units generated by the model had changed the composition of the frontier. Recall that in the original data set all units were enveloped by two facets, Facet D G J and Facet D G H. In the data set generated, all units attained a relative efficiency of 1.0. A similar CFA evaluation of the population generated when the model was asked to maximize both levels of production was performed. Results of this

Table VIII

CFA Relative Efficiency of Organizations Using Estimated Levels of Production and Consumption vs Existing Levels of Production and Consumption

Wings	Upper Bound Estimate of Efficiency Using Current Consumption and Production Levels	Upper Bound Estimate of Efficiency Using Proposed Consumption and Production Levels
A	0.891859	1.000000
B	0.878334	1.000000
C	0.833538	1.000000
D	1.000000	1.000000
E	0.852942	1.000000
F	0.894467	1.000000
G	1.000000	1.000000
H	1.000000	1.000000
I	0.945825	1.000000
J	1.000000	1.000000
K	0.775134	1.000000
L	0.703644	1.000000

evaluation showed that all organizations had reached a rating of 1.0.

The subject of model validation will be addressed again in the next chapter under "Recommendations For Further Research."

Summary of Findings

In this chapter, the resource allocation model was tested. The test case consisted of applying the resource allocation methodology to 12 fictitious tactical fighter wings each consuming two inputs and producing two outputs.

The resource allocation procedure consisted of conducting a Constrained Facet Analysis evaluation of the organizations in the test case for the purpose of detecting relative inefficiencies in the way these organizations produce their outputs. Once the efficiency evaluation was conducted and the marginal rates of productivity for each of the organizations was determined, the resource allocation model was applied assuming a desired increase in production of 5% for both of the goals. Then the model was asked to maximize both outputs.

For validation of the resource allocation model, a Constrained Facet Analysis was performed with a reference set consisting of 13 organizations, each organization generated by the model with those that were in existence before reallocation. The analysis showed a substantial

improvement in the efficiency of operation of the proposed allocations over the existing set of organizations.

There were many simplifying assumptions made about the organizations in the test case. Specific recommendations for further testing of the resource allocation model will be given in the next chapter.

Research Objective 1. Research Objective 1 was to develop specific management techniques that exploited the management information resulting from Constrained Facet Analysis. That objective was met with the resource allocation model described in Chapter III, and demonstrated by the test case in Chapter IV.

Research Objective 2. A secondary objective of this research was to define and explain the relationships between efficiency, effectiveness, productivity, resource allocation and capability. Objective 2 was met in Chapter II through a literature review.

Research Objective 3. The final objective of this research was to explain the types of managerial decisions that can be supported by the application of Data Envelopment Analysis and Constrained Facet Analysis efficiency evaluation methodologies. This objective was met through an explanation of prior applications of DEA and CFA in the literature review, an explanation of these models in Appendix A, and finally through the application of the resource allocation methodology to a test case in Chapter IV.

V. Recommendations for Further Research

Introduction

The methodology for resource allocation described by this research and applied to a test case consisted of allocating resources based on rates of production derived from observed data. An assumption was made for the purpose of illustrating the resource allocation technique that resources could be transferred at no cost if it would increase the overall levels of production (create productivity growth). Additionally, the validation of the model consisted of performing a CFA relative efficiency evaluation of the proposed resource allocations and the existing organizations in the test case to check for productivity growth. The issues of cost and model validation require further research.

Incorporating The Cost Factor Into the Resource Allocation Model

The cost of transferring resources among units is an important consideration when contemplating resource allocation or reallocation. One approach for taking into account the cost of moving resources between organizations would be to run the resource allocation technique developed in this study without considering costs to determine what

levels of resources would be required at each location to meet the specific goals; then run a "transportation" model to minimize the total cost of moving resources among units.

For example, the model determines that in order to achieve a growth of 5% in productivity Wing A is required to increase its consumption of resource X_2 by 2738.914 units, while Wing B must receive an additional 492 units of resource X_1 and also increase its consumption of resource X_2 by 125.567 units, . . ., <u>etc</u>. If the different costs of moving one unit of resource X_1 from Wing A to Wings B through L are known, then it may be possible to set up the distribution of those resources as a minimum cost "transportation" problem that recognizes demands, supplies, and specific costs of moving supplies from alternative locations to the demand points. However, if the minimum total cost of transferring the resources between organizations is excessive, perhaps alternative efficient distributions of resources should be generated. By explicitly considering the costs of transferring resources, management would gain a clearer understanding of what impacts an increase in production would have on costs.

<u>Further Validation of the Resource Allocation Model</u>

The model should be field tested using real data and the expertise of knowledgeable managers. This study applied a new model formulation to a <u>hypothetical</u> set of

organizations. Modern military organizations are far more complex than the test case example; therefore, it would be beneficial to test the model further by applying the resource allocation methodology to an actual set of military organizations and by consulting with knowledgeable managers to see if the allocations suggested by the model are feasible. Time constraints precluded field testing of this model during this research; however, model validation in a real world setting is still needed.

Final Remarks

This research did not explore all possible ramifications of using the management information generated by Constrained Facet Analysis in solving the resource allocation problem. However, the author believes that the resource allocation approach presented in this research promises to provide management with a tool which would help answer the question of where the next defense dollar should be spent to gain greater combat capability.

APPENDIX A: Data Envelopment Analysis and Constrained Facet Analysis

The DEA model evaluates the relative efficiency of organizations by taking into account all observed inputs (resources) and all observed outputs in determining an efficiency rating for each organization. The model expresses the efficiency measure as the ratio of the sum of the weighted outputs to the sum of the weighted inputs, where the weights are assigned by the model to achieve the highest possible efficiency rating for each organization being evaluated.

The model can be converted to an equivalent ordinary linear program using Charnes' theory of fractional linear programming [5:432]. This equivalence is reached by setting the sum of the total weighted inputs equal to one and then requiring that the sum of the weighted outputs be less than or equal to one. The model uses linear programming to provide a "new way" for estimating efficiency and detecting sources of inefficiency from observed data; furthermore, the value of the computed weights for each observed input and output are determined so that the unit being evaluated receives the highest possible efficiency rating [5:430-431]. The original Charnes, Cooper, and Rhodes nonlinear, ratio formulation of the DEA model follows:

$$\underline{\text{Maximize}} \quad h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

Subject To

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \; ; \qquad j=1,2,\ldots,o,\ldots,m$$

$$x_i,\ y_r,\ u_r,\ v_i \ > 0 \; ; \quad r=1,\ldots,s \; ; \quad i=1,\ldots,m$$

The y_{rj}, x_{ij} are the observed outputs and inputs of the jth unit, and the u_r, $v_i > 0$ are the "weights" to be determined by the model. The subscript of "o" identifies the unit being rated; and "the indicated maximization, then accords the most favorable weighting that the constraints will allow" [5:430].

The equivalent linear program can be stated as follows:

$$\underline{\text{Maximize}} \quad h_o = \sum_{r=1}^{s} u_r y_{ro}$$

$$\underline{\text{Subject To}} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0$$

for j =1,2,...,o,...,n (all organizations including organization "o" ,the unit currently being evaluated, in the reference set)

$$\sum_{i=1}^{m} v_i x_{io} = 1$$

$$u_r, v_i > 0 \; ; \quad r = 1,2,\ldots,s \; ; \quad i = 1,2,\ldots,m$$

The constraint in which the sum of the weighted inputs for the organization being rated equals one is the constraint that enables the transformation from an intractable nonlinear mathematical program to a comparatively easy to solve linear program [7].

The development of Data Envelopment Analysis was an important breakthrough in efficiency estimation; however, Clark and others [6] found that this method of evaluation

could overestimate the efficiency of outlier organizations. An outlier is an organization that is relatively inefficient as well as one that is dissimilar to the efficient units in the way that it consumes inputs to produce outputs.

Constrained Facet Analysis was developed to avoid overestimating efficiency.

A full and rigorous description of Constrained Facet Analysis can be found in Clark [6]. Clark's research consisted of extending the frontier facet to provide quasi-envelopment of outlier units. For the purpose of this research, the CFA model was selected because when outlier organizations are present CFA usually generates a greater number of nonzero rates of productivity and substitution.

APPENDIX B: Marginal Rates of Productivity and Substitution

Table B.1

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing A

	1. ∂y_{1A} (.0000439)	2. ∂y_{2A} (.0000132)	3. ∂x_{1A} (.0002842)	4. ∂x_{2A} (.0000162)
1. ∂y_{1A} (.0000439)		-3.3258	.1545	2.7099
2. ∂y_{2A} (.0000132)	-.3007		.0464	.8148
3. ∂x_{1A} (.0002842)	6.4738	21.5303		-17.5432
4. ∂x_{2A} (.0000162)	.3690	1.2273	-.0570	

Table B.2

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing B

	1. ∂y_{1B} (.0000629)	2. ∂y_{2B} (.0000190)	3. ∂x_{1B} (.0004076)	4. ∂x_{2B} (.0000232)
1. ∂y_{1B} (.0000629)		-3.3105	.1543	2.7112
2. ∂y_{2B} (.0000190)	-.3021		.0466	.8190
3. ∂x_{1B} (.0004076)	6.4801	21.4526		-17.5690
4. ∂x_{2B} (.0000232)	.3688	1.2211	-.0569	

Table B.3

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing C

	1. ∂y_{1C} (.0000445)	2. ∂y_{2C} (.0000134)	3. ∂x_{1C} (.0002883)	4. ∂x_{2C} (.0000164)
1. ∂y_{1C} (.0000445)		-3.3209	.1544	2.7134
2. ∂y_{2C} (.0000134)	-.3011		.0465	.8171
3. ∂x_{1C} (.0002883)	6.4787	21.5149		-17.5793
4. ∂x_{2C} (.0000164)	.3685	1.2239	-.0569	

Table B.4

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing D

	1. ∂y_{1D} (.0000380)	2. ∂y_{2D} (.0000385)	3. ∂x_{1D} (.0003136)	4. ∂x_{2D} (.0000287)
1. ∂y_{1D} (.0000380)		- .9870	.1212	1.3240
2. ∂y_{2D} (.0000385)	-1.0132		.1228	1.3415
3. ∂x_{1D} (.0003136)	8.2526	8.1455		-10.9268
4. ∂x_{2D} (.0000287)	.7553	.7455	-.0915	

Table B.5

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing E

	1. ∂y_{1E} (.0000218)	2. ∂y_{2E} (.0000221)	3. ∂x_{1E} (.0001797)	4. ∂x_{2E} (.0000165)
1. ∂y_{1E} (.0000218)		- 1.0138	.1213	1.3212
2. ∂y_{2E} (.0000221)	-1.0138		.1230	1.3394
3. ∂x_{1E} (.0001797)	8.2431	8.1312		-10.8909
4. ∂x_{2E} (.0000165)	.7569	.7466	-.0918	

Table B.6

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing F

	1. ∂y_{1F} (.0000394)	2. ∂y_{2F} (.0000399)	3. ∂x_{1F} (.0003249)	4. ∂x_{2F} (.0000298)
∂y_{1F} (.0000394)		- .9924	.1203	1.3121
2. ∂y_{2F} (.0000399)	-1.0127		.1228	1.3389
3. ∂x_{2F} (.0003249)	8.2462	8.1429		-10.9027
4. ∂x_{2F} (.0000298)	.7563	.7469	-.1225	

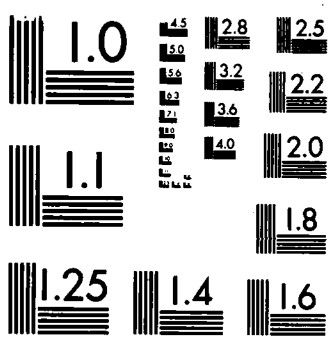
AD-A161 082 DETERMINING OPTIMAL RESOURCEAALLOCATIONS USING PRODUCTION MODELS DERIVED F (U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF SYST 2/2

UNCLASSIFIED J O MONTEMAYOR SEP 85 AFIT/GLM/LSM-85S-53 F/G 15/5 NL

END

FILMED

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Table B.7

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing G

	1. ∂y_{1G} (.0000337)	2. ∂y_{2G} (.0000342)	3. ∂x_{1G} (.0002782)	4. ∂x_{2G} (.0000255)
1. ∂y_{1G} (.0000337)		- .9854	.1211	1.3216
2. ∂y_{2G} (.0000399)	-1.0148		.1229	1.3412
3. ∂x_{2G} (.0002782)	8.2552	8.1345		-10.9098
4. ∂x_{2G} (.0000255)	.7567	.7456	-.0917	

Table B.8

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing H

	1. ∂y_{1H} (.0000589)	2. ∂y_{2H} (.0000597)	3. ∂x_{1H} (.0004856)	4. ∂x_{2H} (.0000445)
1. ∂y_{1H} (.0000589)		- .9866	.1213	1.3236
2. ∂y_{2H} (.0000597)	-1.0136		.1229	1.3416
3. ∂x_{1H} (.0004856)	8.2445	8.1340		-10.9124
4. ∂x_{2H} (.0000445)	.7555	.7454	-.0916	

Table B.9

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing I

	1. ∂y_{1I} (.0000442)	2. ∂y_{2I} (.0000133)	3. ∂x_{1I} (.0002860)	4. ∂x_{2I} (.0000163)
1. ∂y_{1I} (.0000442)		- 3.3233	.1545	2.7117
2. ∂y_{2I} (.0000133)	- .3009		.0465	.8160
3. ∂x_{1I} (.0002860)	6.4706	21.5038		-17.5460
4. ∂x_{2I} (.0000163)	.3688	1.2256	-.0570	

Table B.10

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing J

	1. ∂y_{1J} (.0000379)	2. ∂y_{2J} (.0000114)	3. ∂x_{1J} (.0002454)	4. ∂x_{2J} (.0000140)
1. ∂y_{1J} (.0000379)		- 3.3246	.1544	2.7071
2. ∂y_{2J} (.0000114)	- .3008		.0465	.8143
3. ∂x_{1J} (.0002454)	6.4749	21.5264		-17.5286
4. ∂x_{2J} (.0000140)	.3694	1.2281	-.0570	

Table B.11

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing K

	1. ∂y_{1K} (.0000490)	2. ∂y_{2K} (.0000148)	3. ∂x_{1K} (.0003174)	4. ∂x_{2K} (.0000181)
1. ∂y_{1K} (.0000490)		-3.3108	.1544	2.7072
2. ∂y_{2K} (.0000148)	-.3020		.0466	.8177
3. ∂x_{1K} (.0003174)	6.4776	21.4459		-17.5359
4. ∂x_{2K} (.0000181)	.3694	1.2230	-.0570	

Table B.12

Marginal Rates of Substitution and Marginal Rates of Productivity in the Proper Facet of Wing L

	1. ∂y_{1L} (.0000324)	2. ∂y_{2L} (.0000329)	3. ∂x_{1L} (.0002676)	4. ∂x_{2L} (.0000245)
1. ∂y_{1L} (.0000324)		- .9848	.1211	1.3224
2. ∂y_{2L} (.0000329)	-1.0154		.1229	1.3429
3. ∂x_{1L} (.0002676)	8.2593	8.1337		-10.9224
4. ∂x_{2L} (.0000245)	.7562	.7447	-.0916	

APPENDIX C: The Production Processes of Aircraft Wings in the Test Case

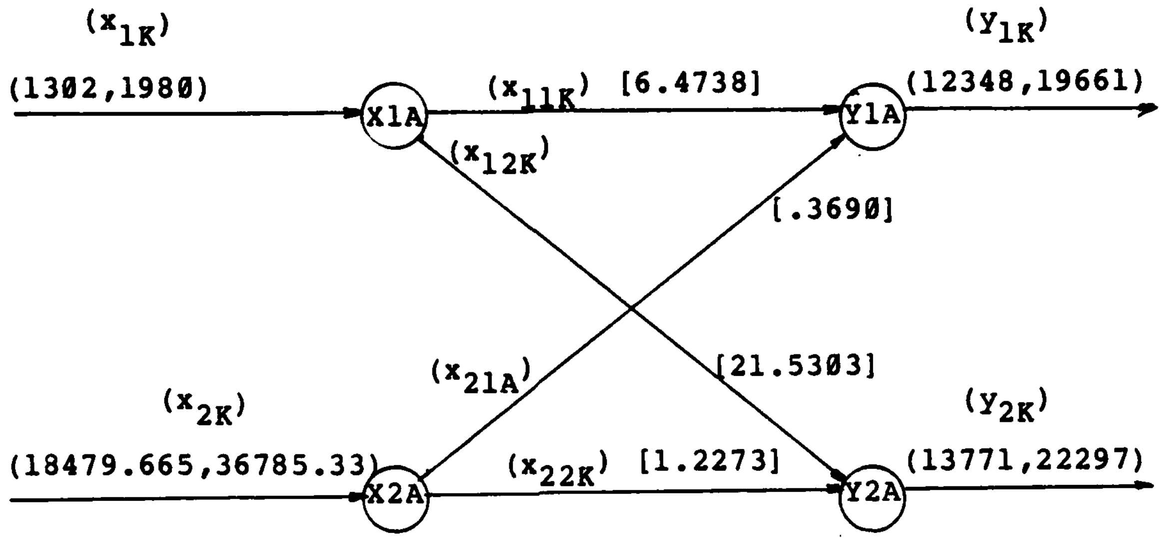


Figure C.1
The Production Process of Wing A

Arc Flow Notation:

( ) - Flow

(min, max) - Flow Bounds

$[P_{irj}]$ - Multiplier

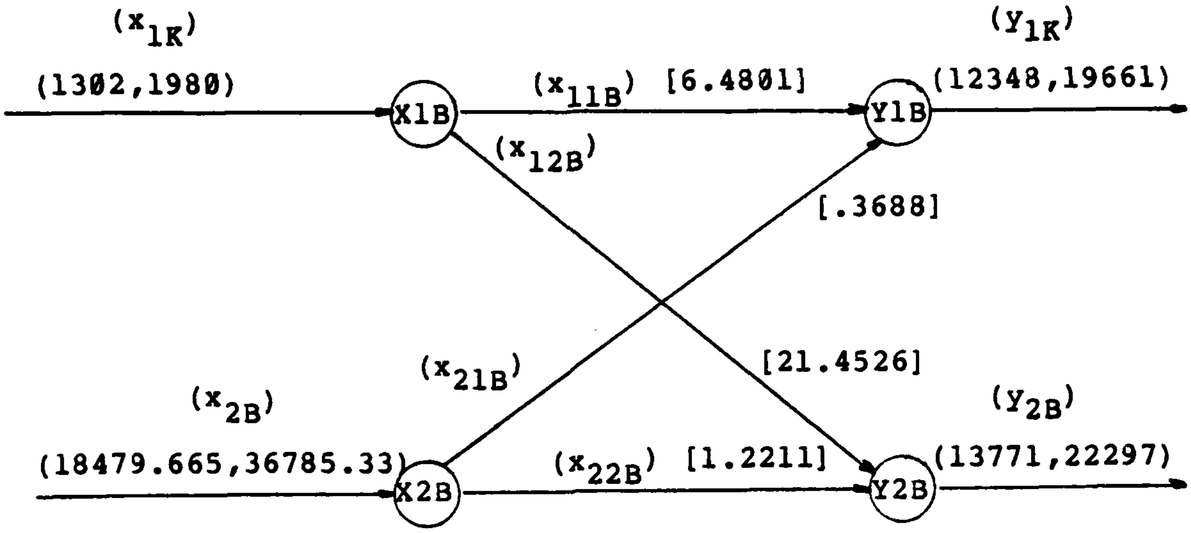


Figure C.2
The Production Process of Wing B

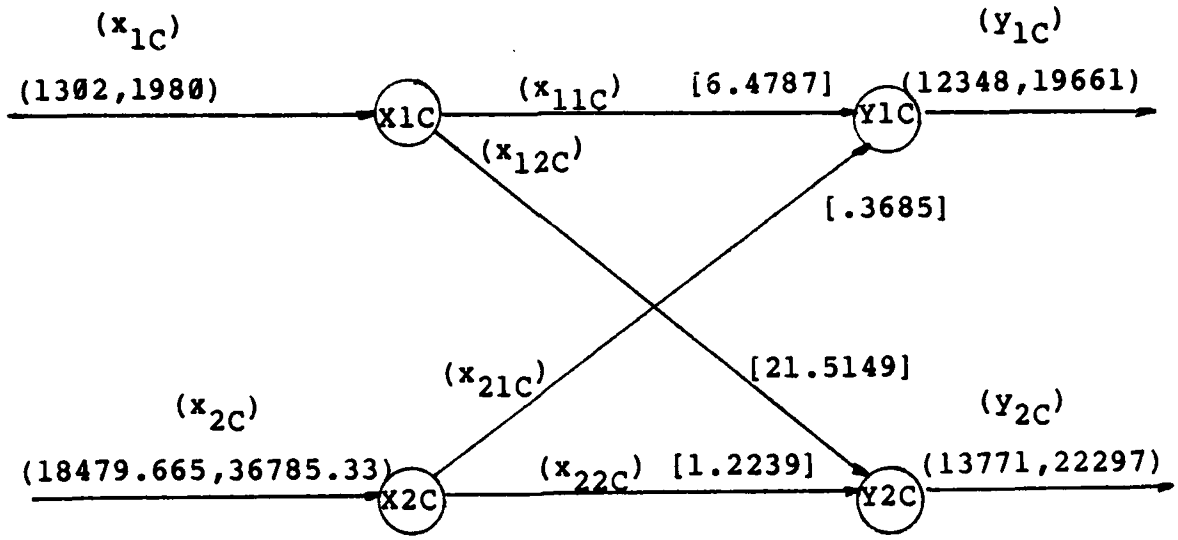


Figure C.3
The Production Process of Wing C

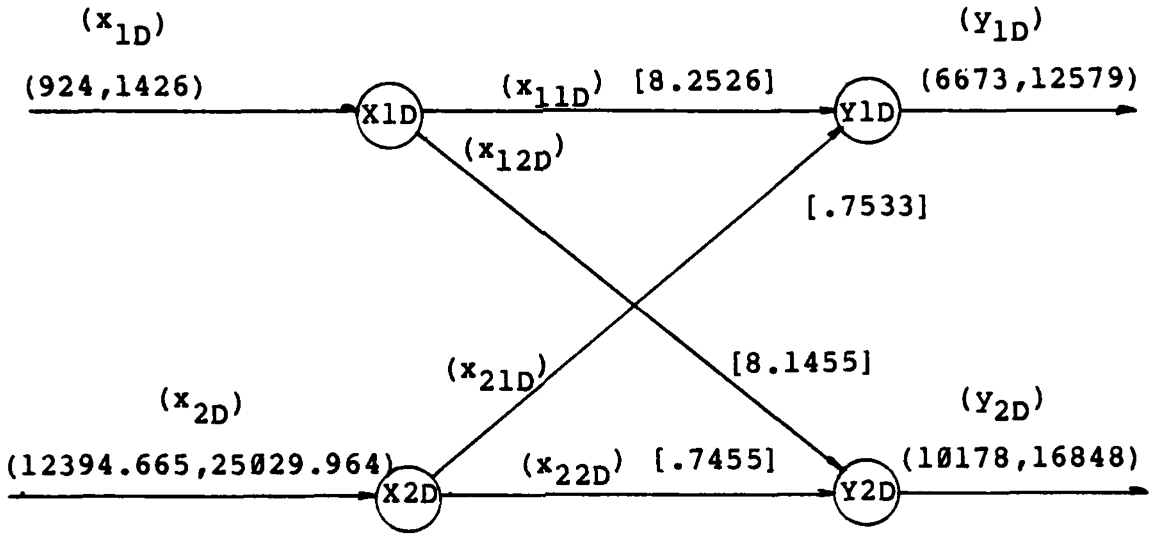


Figure C.4
The Production Process of Wing D

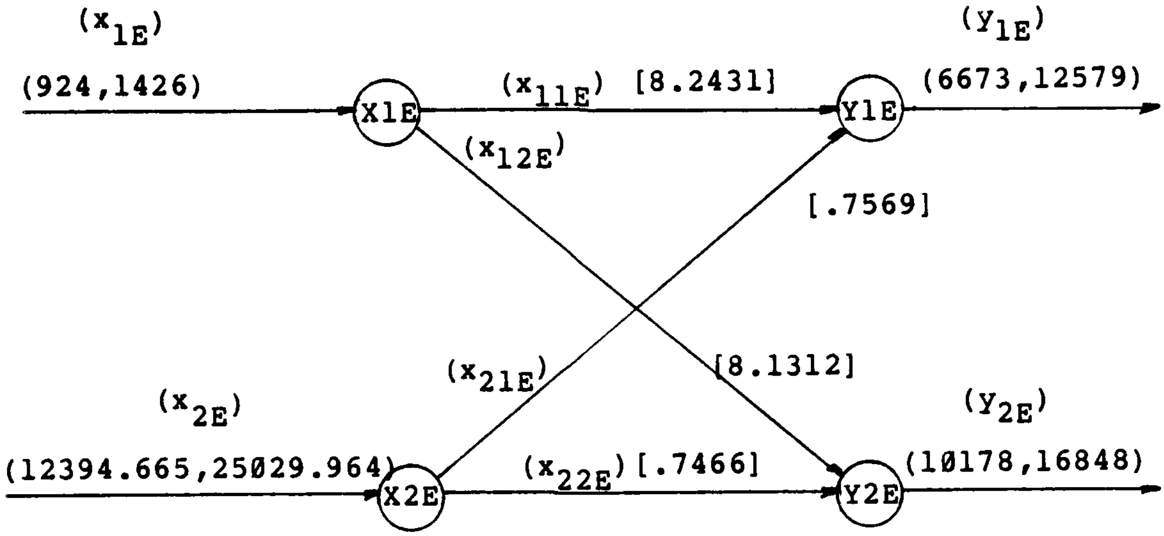


Figure C.5
The Production Process of Wing E

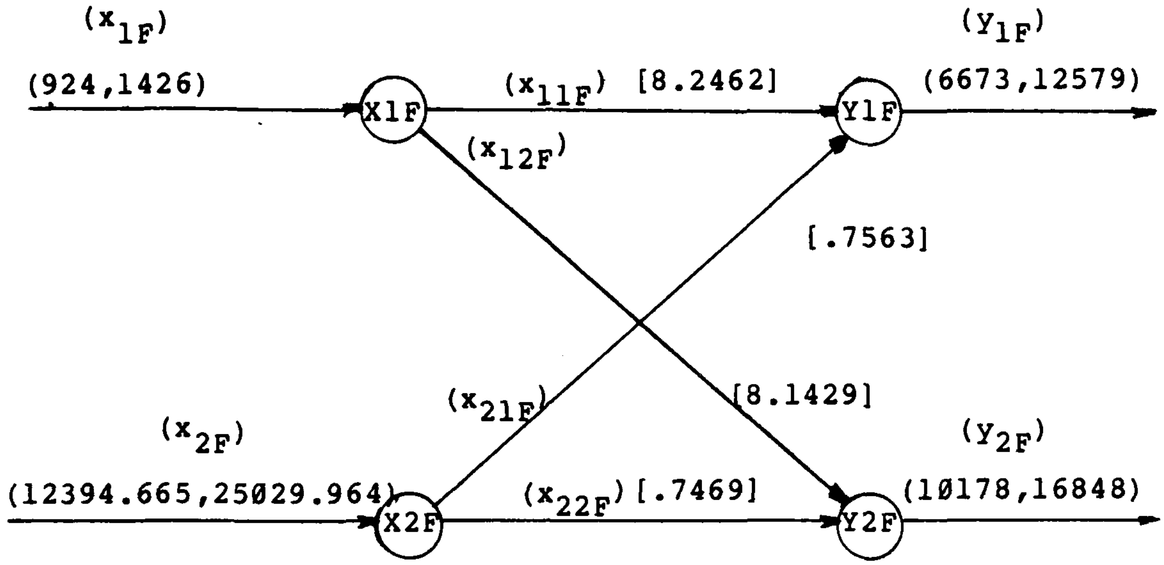


Figure C.6
The Production Process of Wing F

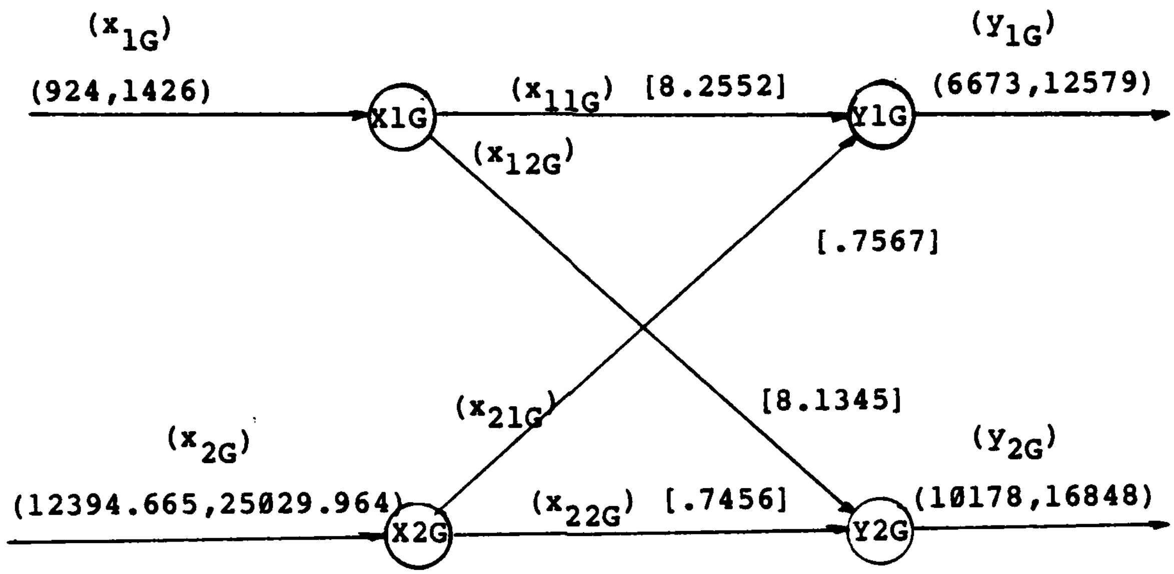


Figure C.7
The Production Process of Wing G

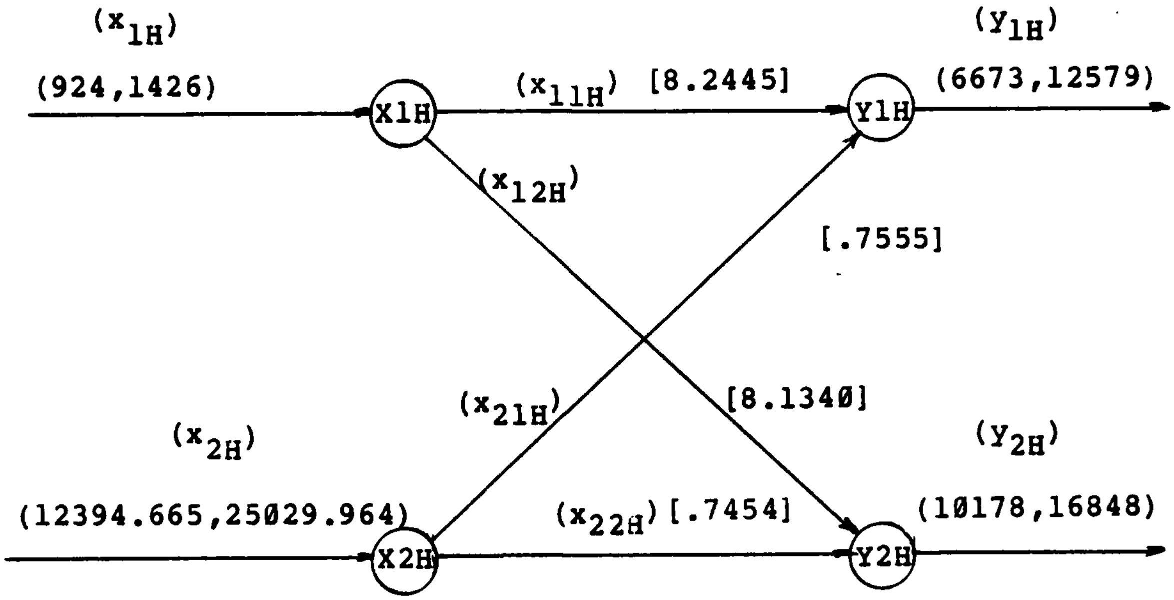


Figure C.8
The Production Process of Wing H

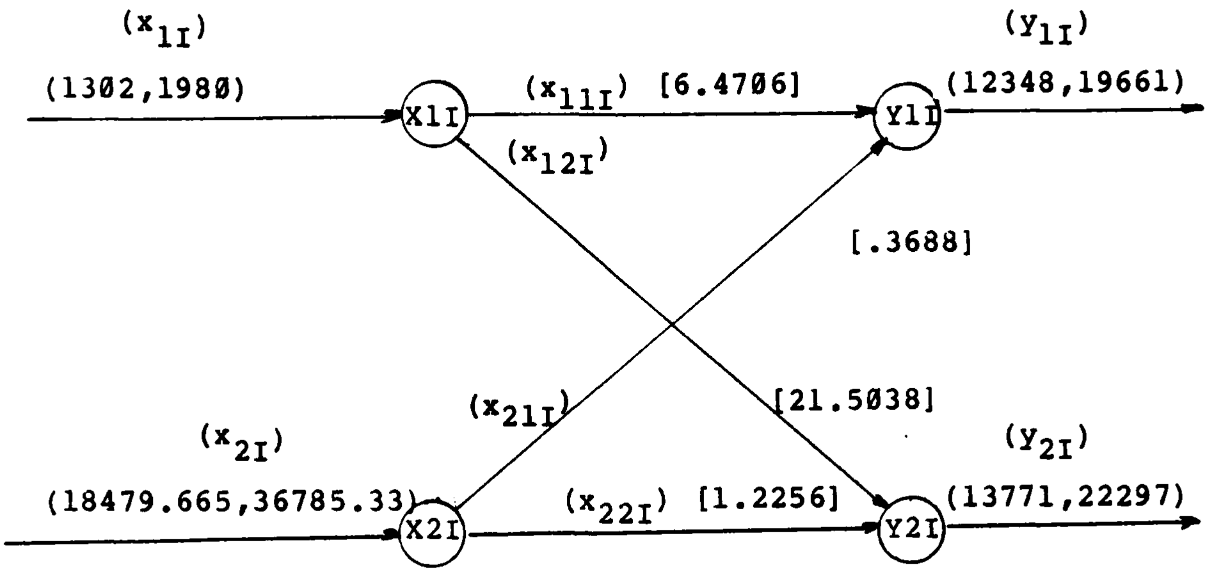


Figure C.9
The Production Process of Wing I

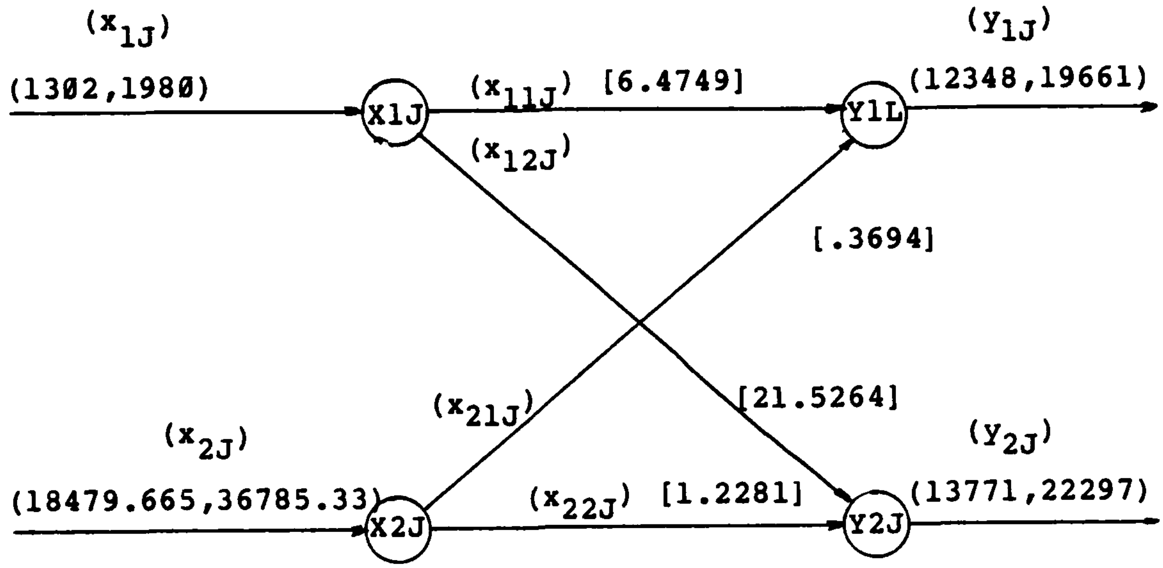


Figure C.10
The Production Process of Wing J

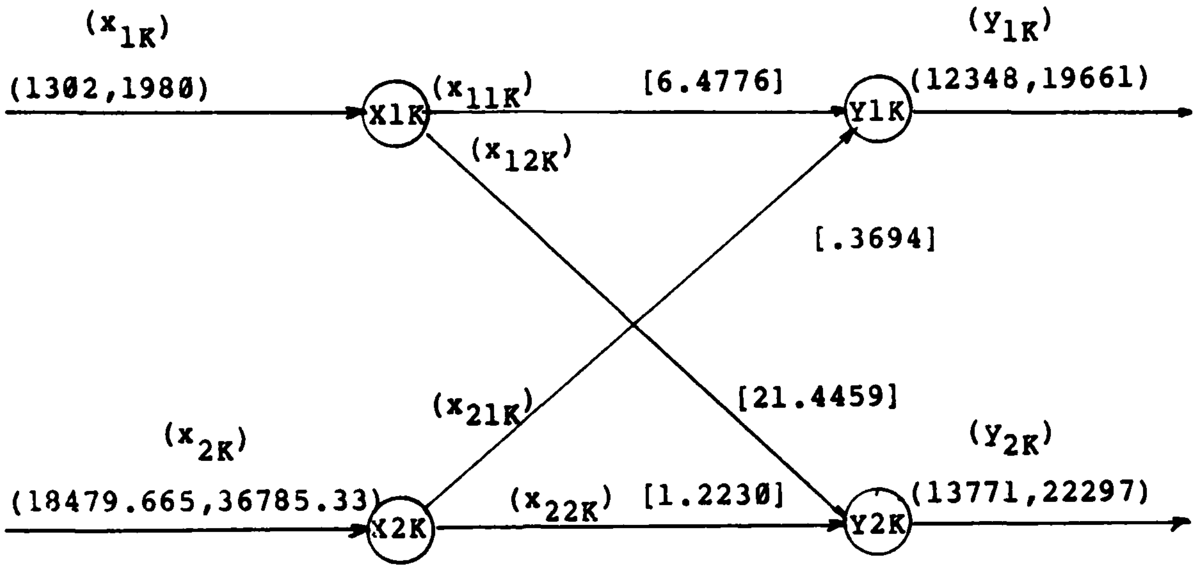


Figure C.11
The Production Process of Wing K

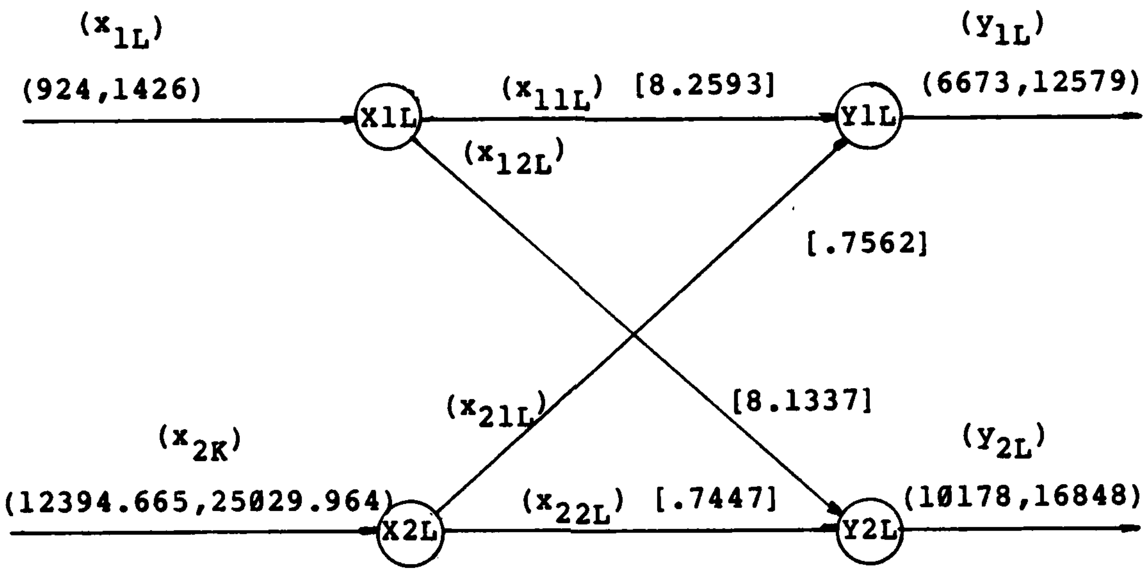


Figure C.12
The Production Process of Wing L

APPENDIX D: Computer Code For Test Case Allocation Network

```
TITLE
THESIS NETWORK:  ALLOCATE RESOURCES GIVEN BUDGET,
EFFICIENCIES, AND EQUAL PREFERENCE FOR THE GOALS

REGULAR

VARIABLES
X1A,X1B,X1C,X1D,X1E,X1F,X1G,X1H,X1I,X1J,X1K,X1L,X2A,X2B,X2C,
X2D,X2E,X2F,X2G,X2H,X2I,X2J,X2K,X2L,X1A1,X1B1,X1C1,X1D1,
X1E1,X1F1,X1G1,X1H1,X1I1,X1J1,X1K1,X1L1,X2A2,X2B2,X2C2,X2D2,
X2E2,X2F2,X2G2,X2H2,X2I2,X2J2,X2K2,X2L2,X1A2,X1B2,X1C2,X1D2,
X1E2,X1F2,X1G2,X1H2,X1I2,X1J2,X1K2,X1L2,X2A1,X2B1,X2C1,X2D1,
X2E1,X2F1,X2G1,X2H1,X2I1,X2J1,X2K1,X2L1,Y1A,Y1B,Y1C,Y1D,Y1E,
Y1F,Y1G,Y1H,Y1I,Y1J,Y1K,Y1L,Y2A,Y2B,Y2C,Y2D,Y2E,Y2F,Y2G,Y2H,
Y2I,Y2J,Y2K,Y2L,S1N,S1P,S2N,S2P

MINIMIZE
S1N + S2N

CONSTRAINTS
X1A+X1B+X1C+X1D+X1E+X1F+X1G+X1H+X1I+X1J+X1K+X1L .EQ.20662
X2A+X2B+X2C+X2D+X2E+X2F+X2G+X2H+X2I+X2J+X2K+X2L .EQ.280949.8
-X1A+X1A1+X1A2 .EQ. 0
-X2A+X2A1+X2A2 .EQ. 0
-X1B+X1B1+X1B2 .EQ. 0
-X2B+X2B1+X2B2 .EQ. 0
-X1C+X1C1+X1C2 .EQ. 0
-X2C+X2C1+X2C2 .EQ. 0
-X1D+X1D1+X1D2 .EQ. 0
-X2D+X2D1+X2D2 .EQ. 0
-X1E+X1E1+X1E2 .EQ. 0
-X2E+X2E1+X2E2 .EQ. 0
-X1F+X1F1+X1F2 .EQ. 0
-X2F+X2F1+X2F2 .EQ. 0
-X1G+X1G1+X1G2 .EQ. 0
-X2G+X2G1+X2G2 .EQ. 0
-X1H+X1H1+X1H2 .EQ. 0
-X2H+X2H1+X2H2 .EQ. 0
-X1I+X1I1+X1I2 .EQ. 0
-X2I+X2I1+X2I2 .EQ. 0
-X1J+X1J1+X1J2 .EQ. 0
-X2J+X2J1+X2J2 .EQ. 0
-X1K+X1K1+X1K2 .EQ. 0
-X2K+X2K1+X2K2 .EQ. 0
-X1L+X1L1+X1L2 .EQ. 0
-X2L+X2L1+X2L2 .EQ. 0
```

```
-6.4738X1A1-.3690X2A1+Y1A  .EQ. 0
-21.5303X1A2-1.2273X2A2+Y2A.EQ. 0
-6.4801X1B1-.3688X2B1+Y1B  .EQ. 0
-21.4526X1B2-1.2211X2B2+Y2B.EQ. 0
-6.4787X1C1-.3685X2C1+Y1C  .EQ. 0
-21.5149X1C2-1.2239X2C2+Y2C.EQ. 0
-8.2526X1D1-.7553X2D1+Y1D  .EQ. 0
-8.1455X1D2-.7455X2D2+Y2D  .EQ. 0
-8.2431X1E1-.7569X2E1+Y1E  .EQ. 0
-8.1312X1E2-.7466X2E2+Y2E  .EQ. 0
-8.2462X1F1-.7563X2F1+Y1F  .EQ. 0
-8.1429X1F2-.7469X2F2+Y2F  .EQ. 0
-8.2552X1G1-.7567X2G1+Y1G  .EQ. 0
-8.1345X1G2-.7456X2G2+Y2G  .EQ. 0
-8.2445X1H1-.7555X2H1+Y1H  .EQ. 0
-8.1340X1H2-.7454X2H2+Y2H  .EQ. 0
-6.4706X1I1-.3688X2I1+Y1I  .EQ. 0
-21.5038X1I2-1.2256X2I2+Y2I.EQ. 0
-6.4749X1J1-.3694X2J1+Y1J  .EQ. 0
-21.5264X1J2-1.2281X2J2+Y2J.EQ. 0
-6.4776X1K1-.3694X2K1+Y1K  .EQ. 0
-21.4459X1K2-1.2230X2K2+Y2K.EQ. 0
-8.2593X1L1-.7562X2L1+Y1L  .EQ. 0
-8.1337X1L2-.7447X2L2+Y2L  .EQ. 0
Y1A+Y1B+Y1C+Y1D+Y1E+Y1F+Y1G+Y1H+Y1I+Y1J+Y1K+Y1L-S1P+S1N
.EQ. 153294.75
Y2A+Y2B+Y2C+Y2D+Y2E+Y2F+Y2G+Y2H+Y2I+Y2J+Y2K+Y2L-S2P+S2N
.EQ. 178038

BOUNDS
X1A,X1B,X1C,X1I,X1J,X1K .GE. 1302
X1A,X1B,X1C,X1I,X1J,X1K .LE. 1980
X2A,X2B,X2C,X2I,X2J,X2K .GE. 18479.665
X2A,X2B,X2C,X2I,X2J,X2K .LE. 36785.33
X1D,X1E,X1F,X1G,X1H,X1L .GE. 924
X1D,X1E,X1F,X1G,X1H,X1L .LE. 1496
X2D,X2E,X2F,X2G,X2H,X2L .GE. 12394.665
X2D,X2E,X2F,X2G,X2H,X2L .LE. 25029.964
Y1A,Y1B,Y1C,Y1I,Y1J,Y1K .GE. 12348
Y1A,Y1B,Y1C,Y1I,Y1J,Y1K .LE. 19661
Y2A,Y2B,Y2C,Y2I,Y2J,Y2K .GE. 13771
Y2A,Y2B,Y2C,Y2I,Y2J,Y2K .LE. 22297
Y1D,Y1E,Y1F,Y1G,Y1H,Y1L .GE. 6673
Y1D,Y1E,Y1F,Y1G,Y1H,Y1L .LE. 12579
Y2D,Y2E,Y2F,Y2G,Y2H,Y2L .GE. 10178
Y2D,Y2E,Y2F,Y2G,Y2H,Y2L .LE. 16848

PRINT

END
```

APPENDIX E: Computer Code Modified for Maximization Routine

```
TITLE
THESIS NETWORK:  MAXIMIZE OUTPUTS GIVEN BUDGET,
EFFICIENCIES, AND EQUAL PREFERENCE FOR THE GOALS

REGULAR

VARIABLES
X1A,X1B,X1C,X1D,X1E,X1F,X1G,X1H,X1I,X1J,X1K,X1L,X2A,X2B,X2C,
X2D,X2E,X2F,X2G,X2H,X2I,X2J,X2K,X2L,X1A1,X1B1,X1C1,X1D1,
X1E1,X1F1,X1G1,X1H1,X1I1,X1J1,X1K1,X1L1,X2A2,X2B2,X2C2,X2D2,
X2E2,X2F2,X2G2,X2H2,X2I2,X2J2,X2K2,X2L2,X1A2,X1B2,X1C2,X1D2,
X1E2,X1F2,X1G2,X1H2,X1I2,X1J2,X1K2,X1L2,X2A1,X2B1,X2C1,X2D1,
X2E1,X2F1,X2G1,X2H1,X2I1,X2J1,X2K1,X2L1,Y1A,Y1B,Y1C,Y1D,Y1E,
Y1F,Y1G,Y1H,Y1I,Y1J,Y1K,Y1L,Y2A,Y2B,Y2C,Y2D,Y2E,Y2F,Y2G,Y2H,
Y2I,Y2J,Y2K,Y2L,Y1,Y2

MAXIMIZE
Y1 + Y2

CONSTRAINTS
X1A+X1B+X1C+X1D+X1E+X1F+X1G+X1H+X1I+X1J+X1K+X1L .EQ.20662
X2A+X2B+X2C+X2D+X2E+X2F+X2G+X2H+X2I+X2J+X2K+X2L .EQ.280949.8
-X1A+X1A1+X1A2 .EQ. 0
-X2A+X2A1+X2A2 .EQ. 0
-X1B+X1B1+X1B2 .EQ. 0
-X2B+X2B1+X2B2 .EQ. 0
-X1C+X1C1+X1C2 .EQ. 0
-X2C+X2C1+X2C2 .EQ. 0
-X1D+X1D1+X1D2 .EQ. 0
-X2D+X2D1+X2D2 .EQ. 0
-X1E+X1E1+X1E2 .EQ. 0
-X2E+X2E1+X2E2 .EQ. 0
-X1F+X1F1+X1F2 .EQ. 0
-X2F+X2F1+X2F2 .EQ. 0
-X1G+X1G1+X1G2 .EQ. 0
-X2G+X2G1+X2G2 .EQ. 0
-X1H+X1H1+X1H2 .EQ. 0
-X2H+X2H1+X2H2 .EQ. 0
-X1I+X1I1+X1I2 .EQ. 0
-X2I+X2I1+X2I2 .EQ. 0
-X1J+X1J1+X1J2 .EQ. 0
-X2J+X2J1+X2J2 .EQ. 0
-X1K+X1K1+X1K2 .EQ. 0
-X2K+X2K1+X2K2 .EQ. 0
-X1L+X1L1+X1L2 .EQ. 0
-X2L+X2L1+X2L2 .EQ. 0
```

```
-6.4738X1A1-.3690X2A1+Y1A   .EQ.0
-21.5303X1A2-1.2273X2A2+Y2A.EQ.0
-6.4801X1B1-.3688X2B1+Y1B   .EQ. 0
-21.4526X1B2-1.2211X2B2+Y2B.EQ.0
-6.4787X1C1-.3685X2C1+Y1C   .EQ. 0
-21.5149X1C2-1.2239X2C2+Y2C.EQ.0
-8.2526X1D1-.7553X2D1+Y1D   .EQ. 0
-8.1455X1D2-.7455X2D2+Y2D   .EQ. 0
-8.2431X1E1-.7569X2E1+Y1E   .EQ. 0
-8.1312X1E2-.7466X2E2+Y2E   .EQ. 0
-8.2462X1F1-.7563X2F1+Y1F   .EQ.0
-8.1429X1F2-.7469X2F2+Y2F   .EQ. 0
-8.2552X1G1-.7567X2G1+Y1G   .EQ. 0
-8.1345X1G2-.7456X2G2+Y2G   .EQ.0
-8.2445X1H1-.7555X2H1+Y1H   .EQ. 0
-8.1340X1H2-.7454X2H2+Y2H   .EQ. 0
-6.4706X1I1-.3688X2I1+Y1I   .EQ. 0
-21.5038X1I2-1.2256X2I2+Y2I.EQ.0
-6.4749X1J1-.3694X2J1+Y1J   .EQ. 0
-21.5264X1J2-1.2281X2J2+Y2J.EQ.0
-6.4776X1K1-.3694X2K1+Y1K   .EQ. 0
-21.4459X1K2-1.2230X2K2+Y2K.EQ.0
-8.2593X1L1-.7562X2L1+Y1L   .EQ. 0
-8.1337X1L2-.7447X2L2+Y2L   .EQ. 0
-Y1A-Y1B-Y1C-Y1D-Y1E-Y1F-Y1G-Y1H-Y1I-Y1J-Y1K-Y1L+Y1 .EQ. 0
-Y2A-Y2B-Y2C-Y2D-Y2E-Y2F-Y2G-Y2H-Y2I-Y2J-Y2K-Y2L+Y2 .EQ. 0

BOUNDS
X1A,X1B,X1C,X1I,X1J,X1K .GE. 1302
X1A,X1B,X1C,X1I,X1J,X1K .LE. 1980
X2A,X2B,X2C,X2I,X2J,X2K .GE. 18479.665
X2A,X2B,X2C,X2I,X2J,X2K .LE. 36785.33
X1D,X1E,X1F,X1G,X1H,X1L .GE. 924
X1D,X1E,X1F,X1G,X1H,X1L .LE. 1496
X2D,X2E,X2F,X2G,X2H,X2L .GE. 12394.665
X2D,X2E,X2F,X2G,X2H,X2L .LE. 25029.964
Y1A,Y1B,Y1C,Y1I,Y1J,Y1K .GE. 12348
Y1A,Y1B,Y1C,Y1I,Y1J,Y1K .LE. 19661
Y2A,Y2B,Y2C,Y2I,Y2J,Y2K .GE. 13771
Y2A,Y2B,Y2C,Y2I,Y2J,Y2K .LE. 22297
Y1D,Y1E,Y1F,Y1G,Y1H,Y1L .GE. 6673
Y1D,Y1E,Y1F,Y1G,Y1H,Y1L .LE. 12579
Y2D,Y2E,Y2F,Y2G,Y2H,Y2L .GE. 10178
Y2D,Y2E,Y2F,Y2G,Y2H,Y2L .LE. 16848

PRINT

END
```

Bibliography

1. Bessent, Authella M., E. Wailand Bessent, Lieutenant Colonel Charles T. Clark, and Joyce Elam. "Constrained Facet Analysis, A New Method For Evaluating local Frontiers of Efficiency and Performance." Air Force Journal of Logistics, 8, (3): 2-8 (Summer 1984).

2. Bessent, Authella M., E. Wailand Bessent, and Lieutenant Colonel Charles T. Clark "Specification of Inputs and Outputs in Data Envelopment Analysis (DEA)." Working paper. Educational Productivity Council. The University of Texas, Austin TX, 1983.

3. Byers, Timothy A., and Susanne M. Waylett. Measurement of Air Force Fire Department Productivity: An Evaluation of Efficiency/Effectiveness. MS Thesis, AFIT/GEM/LSM/84S-4. School of Systems and Logistics, Air Force Institute of Technology (AU), Wright Patterson AFB OH, September 1984. (AD-A146-874)

4. Charnes, A., W. Cooper, and E. Rhodes. "Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through." Management Science, 27, 6(668-697), 1981.

5. -----. "Measuring the Efficiency of Decision Making Units." The European Journal of Operations Research, 2(429-444), 1978.

6. Clark, C. T. Data Envelopment Analysis and Extensions for Decision Support and Management Planning. Unpublished PhD thesis, The University of Texas at Austin, Austin TX, 1983.

7. Clark C. T., Lecture materials distributed in LM6.35, Evaluating the Efficiency, Productivity and Performance of Air Force Organizations. School of Systems and Logistics, Air Force Institute of Technology (AU), Wright Patterson AFB OH, January 1985.

8. Department of the Air Force. Air Force Productivity Enhancement Program (PEP). AFR 25-3. Washington: HQ USAF, 25 February 1982.

9. Department of Defense. Productivity Enhancement, Measurement, and Evaluation--Operating Guidelines and Reporting Instructions. DODI 5010.34, Washington: Government Printing Office, 4 August 1975.

10. Farrell, M. J., "The Measurement of Productive Efficiency." Journal of the Royal Statistical Society, 120 3(253-290), 1957.

11. Glover, F., J. Hultz, D. Klingman, and J. Stutz. "Generalized Networks: A Fundamental Computer-Based Planning Tool." Management Science, 24(1209-1220), August 1978.

12. Glover, Fred, Gene Jones, David Karney, Darwin Klingman, and John Mote. "an Integrated Production, Distribution, and Inventory Planning System." Interfaces, 9, 5(21-35), November 1979.

13. Glover, F., and D. Klingman. "Network Applications in Industry and Government." AIIE Transactions, 9, 4(363-373), December 1977.

14. Gonnerman Valerie J. Performance Evaluation of A-10 Aircraft Maintenance Units and Aircraft Using Constrained Facet Analysis. MS Thesis, AFIT/GLM/84S-26. School of Systems and Logistics, Air Force Institute of Technology (AU), Wright Patterson AFB OH, September 1984. (AD-A146-955)

15. Gould, J. P., and C. E. Ferguson. Micro Economic Theory. (fifth edition). Homewood IL: Richard D. Irwin inc., 1980.

16. Hitt, R. E., and R. F. Horace. Feasibility of Measuring Technical Productivity Improvements in AFLC Depot-Level Maintenance Using the DATA Envelopment Analysis(DEA) and Constrained Facet Analysis (CFA) Models. MS Thesis, AFIT/GLM/LS/84S, School of Systems and Logistics, Air Force Institute of Technology (AU), Wright Patterson AFB OH, June 1984. (AD-A147-329)

17. Jensen, P. A., and W. J. Barnes. Network Flow Programming. New York: John Wiley and Sons, 1980.

18. Korb, Lawrence J., Assistant Secretary of Defense for Manpower, Installations, and Logistics. "The Four Components Of Combat Capability." Address to AFIT students, Air Force Institute of Technology (AU), Wright Patterson AFB OH, July 1983.

19. Lewin, A. Y., R. C. Morey, and T. J. Cook. "Evaluating the Administrative Efficiency of Courts." Omega, 4, 10(401-411), 1982.

20. Sherman, H. Measurement of Hospital Efficiency Using Data Envelopment Analysis. Unpublished DBA Thesis, Graduate School of Business, Harvard University, Cambridge MA, 1981.

21. Simon, H. A., Administrative Behavior: A Study of Decision Making Processes in Administrative Organizations. New York: The Macmillan Co., 1976.

Vita

Captain Jose O. Montemayor was born on 29 January 1951 in Brownsville, Texas. He graduated from the Monterrey Institute of Technology preparatory school in Monterrey, Mexico, in 1968. He enlisted in the Air Force in March 1969 and served first in aircraft maintenance and later as a construction manager with Base Civil Engineering. While enlisted, he continued his education and received a Bachelor of Science Degree from St. Edwards University in Austin, Texas, in August 1975.

He was commissioned in the Air Force in December of 1978 and was assigned to the 381th Strategic Missile Wing (SMW) in McConnell AFB, Kansas. While in Kansas, he attended Webster University and received the degree of Master of Arts. At the 381th SMW, he served as a Missile Combat Crew Commander and later as a Missile Operations Staff Officer until entering the Graduate School of Systems and Logistics, Air Force Institute of Technology, in May 1984.

Permanent Address: 1426 W. Levee
Brownsville, Texas 78520

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

AD-A161082

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GLM/LSM/85S-53	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION School of Systems and Logistics	6b. OFFICE SYMBOL (If applicable) AFIT/LSM
7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State and ZIP Code) Air Force Institute of Technology Wright-Patterson AFB, Ohio 45433	7b. ADDRESS (City, State and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)
9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT NO.
11. TITLE (Include Security Classification) See Box 19	
12. PERSONAL AUTHOR(S) Jose O. Montemayor, B.S., M.A., Capt, USAF	
13a. TYPE OF REPORT MS Thesis	13b. TIME COVERED FROM ______ TO ______
14. DATE OF REPORT (Yr., Mo., Day) 1985 September	15. PAGE COUNT 122
16. SUPPLEMENTARY NOTATION	

17. COSATI CODES		
FIELD	GROUP	SUB. GR.
15	05	
12	02	

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Resource Management, Production Models, Efficiency, (CFA) Constrained Facet Analysis, Network Analysis

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Title: DETERMINING OPTIMAL RESOURCE ALLOCATIONS USING PRODUCTION MODELS DERIVED FROM EFFICIENCY ANALYSIS OF EMPIRICAL DATA

Thesis Advisor: Charles T. Clark, Lt Col, USAF
Head, Department of Logistics Management

Approved for public release: IAW AFR 190-1.
LYNN E. WOLAVER 11 Sep 85
Dean for Research and Professional Development
Air Force Institute of Technology (ATC)
Wright-Patterson AFB OH 45433

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Charles T. Clark, Lt Col, USAF	22b. TELEPHONE NUMBER (Include Area Code) 513-255-5023
22c. OFFICE SYMBOL AFIT/LSM	

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

Evaluating the performance of nonprofit organizations and formulating resource allocation policy has long been recognized as a difficult problem for management to solve. This ~~research~~ thesis defines the relationships between the terms efficiency, effectiveness, productivity, resource allocation, and capability as it pertains to military organizations. This research studied possible ways in which the recently developed efficiency measurement methodology, Constrained Facet Analysis, might be used in solving the resource allocation problem.

The approach taken was that of experimentation with a resource allocation model using a data set that simulated a group of 12 tactical fighter wings each using 2 types of resources, manpower and materiel, and producing 2 types of outputs, sorties and mission capable aircraft days.

The resource allocation model consisted of a generalized network model. Networks have graphic properties which make possible the presentation of the resource allocation problem in nonmathematical terms. Furthermore, the translation of the graphic network model into a mathematical program for computer solution is relatively easy.

The methodology pursued by this research consisted of experimentation with the two-input, two-output case; i.e., given that relative efficiencies and apparent rates of productivity can now be measured among a group of related organizations, should available resources be allocated to increase production to some set level? Or, what is the maximum level of production that can be expected?

The research concludes with recommendations for field testing the resource allocation model using actual data and the help of knowledgeable managers.

END

FILMED

12-85

DTIC